Effective Endowment, Trade and Wages

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Abstract

This paper presents a model that incorporates multidimensional skill endowment for each agent and team production where team members completely specialize in different tasks into the standard Heckscher-Ohlin framework, and investigates their effects on trade and wages. The equilibrium is characterized by the “effective endowment”, the part of endowment that is actually utilized in production, which depends on the team matches and the task specialization within matched teams. The paper shows that: (1) the endowment correlation between skill dimensions for each agent and the skill dispersion across agents, additional to aggregate endowment, both matter for the patter of specialization; (2) the different endowment distributions also generate different wage inequality across countries; a common job polarization patter is generated in all developed economies in the globalization era; (3) there are new gains from trade, attributed to potential adjustments of the effective endowment after integration. It provides an unifying framework to explain both the trade patterns and labor market outcomes between similar countries. It also reveals a new channel through which institutions may have effects on comparative advantage and trade. In particular, the effects of different educational policies and labor market institutions on trade through shaping the skill distributions in each country are highlighted. Moreover, by linking globalization to the labor market, it provides an alternative explanation for some stylized facts on wage inequality and employment changes.

Keywords: multidimensional endowment, team production, effective endowment, comparative advantage, gains from trade, job polarization, wage inequality

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1 Introduction

During the era of globalization, the United States (US) and (continental) Europe have experienced different trade patterns despite similar aggregate endowment. In general, it seems that the European countries export relatively more manufacturing goods, cars and equipments for instance, while the US mainly exports services such as software and consulting.\textsuperscript{1} The labor market outcomes in these two regions also differ.\textsuperscript{2} It is well known that the wage inequality is higher in the US than in Europe. Recent literature finds a common “job polarization” pattern of employment change in both regions, accompanying the divergent wage evolutions.\textsuperscript{3}

The current literature still lacks a unified framework to explain both the behavior of trade and labor markets in these two regions in the globalized economy.\textsuperscript{4} This paper fills this gap by building a model that can explain both the patterns of trade and the labor market outcomes in the US and Europe, based on the observation that the distributions of skills across the labor forces of the two regions differ.\textsuperscript{5}

The model deviates from the standard Heckscher-Ohlin framework by assuming a multidimensional skill endowment for each agent and team production in which team members completely specialize in different tasks.\textsuperscript{6} In the optimal labor allocation, agent matches and task assignments within each match are chosen to maximize the total output. As a result, the equilibrium is characterized by the “effective endowment”, the part of endowment that is actually utilized in production, instead of the initial skill endowments. Under these settings, it is then shown that both the endowment correlation between skill dimensions and the skill dispersion across agents matter for the pattern of specialization and wages, complementing

\textsuperscript{1}This is true especially for Germany. The share of service in total export also decreases for France and Italy relative to the US during this period. Data source: World Bank and OECD national accounts data.

\textsuperscript{2}See Freeman and Katz (1995) among others for reviews of these differences. This paper does not explain all these differences. Instead, it focuses on explaining the trade patterns and the coexistence of a job polarization in both the US and Europe with divergent wage evolutions across the two regions.

\textsuperscript{3}Job polarization is a pattern of employment change in which the top-wage jobs and bottom-wage jobs experience a higher increase than the middle-wage jobs. See Autor, Katz and Kearney (2006, 2008), Goos and Manning (2007) and Goos, Manning and Salomons (2009) for detail. It is also known as the “shrinking middle class” phenomenon in the news and media.

\textsuperscript{4}Within the trade literature, the effect of trade integration on inequality is inconclusive; see Goldberg and Pavcnik (2007) and Harrison, McLaren and McMillan (2011) for reviews of related works. In the labor literature, the effect of trade and off-shoring on labor outcomes is usually considered quantitatively small. Particularly, when explaining the job polarization with the task approach, Autor et al (2006, 2008) argues that the off-shoring ability of different tasks only has a quantitatively small effect.

\textsuperscript{5}See Grossman and Maggi (2000), Bombardini et al (2012), and Ohnsorge and Trefler (2007) for evidences on the differences of talent distributions across countries.

the standard channel in which relative aggregate factor endowments shape the comparative advantage.

Upon integration, there is a new source of gains from trade in this model. These new gains are attributed to potential adjustments of the effective endowment through reshuffling the production teams or reassigning tasks within teams. Traditional gains from trade originate from the relative price changes and the reallocation of resources into industries where one has comparative advantages. This model does not take current utilized resources, the effective endowment, as given. Instead, it further allows the economy to adjust its utilized resources in production so as to achieve better outcome from its initial endowment after integration.

When applied to analyse the labor markets in the US and Europe, this model links globalization with the wages and employment changes. It provides an alternative explanation for the job polarization pattern found in both regions along with different wage evolutions. More generally, this model generates a universal job polarization in all the North economies when they open to trade with the South in the global economy. Meanwhile, divergent wage evolutions across North countries result from their different comparative advantages and specialization patterns among themselves, attributed to their different endowment correlations and skill dispersions.

To see the intuition behind these results, we first start with the endowments. The initial skill endowments of agents are shaped by the educational systems, which are exogenously given in each country. Think of agents’ skills in two dimensions: the entrepreneur’s managerial ability and the worker’s production skill. Correspondingly, output production requires a manager task and a worker task to be performed, with different task intensities in different industries. In the competitive environment, each agent on the team will get his share of contribution to the final output, which is equal to the intensity of the task that he performs. Obviously, those industries with extreme-value intensities have higher wage inequality.

In an educational system where general-skill training is emphasized, in the US for example, there is a high correlation between the two dimensions in each agent’s skill endowment. On the other hand, in an educational system where skill-specific vocational training is emphasized, in continental Europe for instance, the correlation between the two skill dimensions in each agent’s endowment is low. Assuming similar aggregate educational resources

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7 I will defend this assumption by arguing that the effect of changes in the educational system on skill endowments takes a long period of time to be realized. Thus skill endowment seems more like an exogenous variable than others in this model, such as the globalization/integration.

8 While one can think of other kinds of skill dimensions, it is not crucial what and how many the skill dimensions are. The key is that with completely specialized agents in team production, individuals face a trade-off between their skill attributes.

9 A high correlation between two endowment attributes for agents means that if he/she is a good manager, he/she has a high probability to be also a good worker.
(aggregate skill endowment) along each dimension in these two regions, their endowment distributions still differ in their endowment correlations due to different educational policies. When production is organized in teams, the high-correlation US ends up with more teams with extreme-value effective skill-ratios in production. These teams have the comparative advantage in those industries with extreme-value task intensities. In the competitive environment this results in higher wage inequality in the US than in Europe. In reality, service industries are usually those where some star member’s exceptional work largely determines the output, while other supporting members only contribute a small share. For example, in the software industry, the manager is in charge of problem-solving task, while the worker is just typing and copying the codes. The value is mainly attributed to the manager’s performance. Hence he will get a very large share of the final output. On the other hand, manufacturing industries usually needs both tasks to be performed with similar intensity. For example, when producing a car, it is very important for the managers to know how to organize the production; it is also important for workers to do a good job in actually producing the car. In the end, higher endowment correlation results in comparative advantage for the US in those extreme-intensity service industries and higher wage inequality level; the opposite is true for the Europe. Moreover, when two countries have the same aggregate skill endowment along each dimension and the same correlation, a higher skill dispersion along one or more dimensions also generates more teams with extreme-value effective skill-ratios in production. Thus a higher dispersion amplifies the effect of a higher correlation on the comparative advantage and wage inequality.

In the case of North-South trade, assuming that South countries have caught up with the North in their worker training but not in their entrepreneur training, the relative endowment of entrepreneurial ability is lower in the South. The existing industries in the North are biased in their task intensities such that the managerial task is more important and intensively used in all industries. Thus before globalization, the North economies have their

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10 See Krueger and Kumar (2004a) for a comparison of educational systems in the US and European countries. See Ohnsorge and Trefler (2007) for evidences on skill correlation differences.

11 Effective skill-ratio is the ratio of the two utilized skills from the team-members.

12 See Ohnsorge and Trefler (2007) for more on the comparative advantage for endowment bundles with different skill-ratios. In general, the intuition is that teams choose the optimal industry according to their (effective) skill ratios, to maximize the output of the whole (effective) bundle.

13 This result is also shown under non-competitive environment in the text.

14 Indeed, the dispersion (or diversity) of human capital/skill distribution is found to be higher in the US than those continental European countries. See Grossman and Maggi(2000), Bombardini et al (2012) for some primary evidence.

15 This is likely true due to the lack of entrepreneurial culture, lower financial development or poor institutions in developing countries.

16 Given the industry structure in the North, with very small share of agriculture, I argue that this assumption is reasonable.
top-wage jobs and bottom-wage jobs in those service industries with very high managerial
task intensity and very low worker-task intensity. After integration with the South, the
North economies have comparative advantage in these service industries. Hence there are
more teams with extreme skill-ratios formed and entering the service industries. As a result,
there are higher increases in employment for those top-wage and bottom-wage jobs relative
to those middle-wage jobs in the North. Thus this model can generate an universal pattern
of job polarization in all the North countries as empirically found.\footnote{When combined with the trade between North economies, this model predicts that the job polarization pattern is more significant in the US than in Europe. Current literature still lacks rigorous investigation on this comparison. However, the data and figures from Autor et al (2006) and Goos and Manning (2007) seems to indicate that the extent of job polarization is indeed greater in the US than in Europe.}

As contributions to the trade and labor literature, this paper points out the effect of
educational policies on the comparative advantage by showing that the higher moments of
endowment distributions shaped by a country’s educational system are crucial in trade; By
providing an alternative explanation for the job polarization from an open economy per-
pective, it also indicates a potential role for globalization behind those labor market phe-
nomenons. Hence the interdependence between trade and labor market is also highlighted.
In particular, this paper is closely related to several strands of literature as follows.

First, this paper extends the standard channel in which relative factor endowments shape
the pattern of trade by showing that higher moments of the endowment distribution also
matter. Grossman and Maggi (2000) (henceforth GM) first put forward the idea that a
relatively diverse skill distribution generates comparative advantage in industries with sub-
modular technologies. Bombardini et al (2012) find empirical support for GM’s idea. Ohn-
sorge and Trefler (2007) (henceforth OT) further develop this idea in an open economy model
with multidimensional skill endowment. This paper shares a lot of implications with OT.
The difference is the team production assumption, which allows me to draw new implications
on the job polarization and wage inequality in the US and Europe.\footnote{See also Bougheas and Riezman (2007) and Sly (2012) for more about the effect of endowment distributions on trade patterns.}

Second, this paper complements those works on the gains from trade by identifying a
new source of gains from trade. These new gains come from the adjustments of the effective
endowment upon integration.\footnote{There are also papers emphasizing the change of utilized endowment after trade liberalization, due to the increased market competition for example. The under-utilization of initial endowment results from various distortions. However, in my model, the autarky utilization of endowment is the optimal choice. Thus the new gains from trade in this model exist even when the market is complete.} There is a large number of papers along this line, see Broda and Weinstein (2006), Feenstra (2004) for example. Moreover, this model may exhibit gains similar with those in the scale economies in certain circumstances.

Third, this paper also extends the Roy model by showing that different endowment
structures, higher correlation and dispersion in the US than in Europe, contribute to the inequality differences. The trade between these two regions amplifies this difference. Gould (2002) finds that the inequality increase in the US is increasingly characterized by the absolute advantage effect, indicating a decrease of the comparative advantage effect. Blum (2008) finds that the increasing share of the service sector in the US explains almost 60% of the relative increase in wages of skilled workers between 1970 and 1996. On the other hand, this paper also indicates a role for globalization behind the job polarization pattern, complementing the existing explanations proposed by Autor et al. (2006, 2008), Autor and Dorn (2012) among others.\footnote{\text{For other explanations for job polarization, see Costinot and Vogel (2010), Monte (2011); For more on the interdependence between trade and labor market, see Davidson and Sly (2012) for an example. The importance of team production organization is also highlighted there.}}

Fourth, this paper shares some implications with several other papers. Eeckhout and Jovanovic (2012) show that there are more managers in the developed countries, which is a natural result of this model when task off-shoring is allowed. Krueger and Kumar (2004a, 2004b) also highlight the effect of educational policies. Instead of focusing on the effects on technology adoption, hence growth rate, I focus on their comparative static effects on the pattern of specialization and wages. From a development point of view, this paper also shares with Buera and Kaboski (forthcoming) and Kaboski (2009) in explaining the increasing skill premium (or wage inequality) in the US with the structural changes towards service industries.

The next section presents a baseline model, illustrating the basic structure and main intuitions. The general model with continuous endowment levels and a continuum of industries is laid out in section 3. The generalized trade theories, the new gains from trade and the implications on wage inequality and job polarization are presented after the general model. Section 3 ends with some other model applications and discussions. Section 4 concludes.

2 The Baseline Model

2.1 Setup

2.1.1 Endowment

In the baseline model I consider the two-dimensional talents endowment, think of them as the entrepreneurial ability and worker skills: \((E, W)\). Accordingly in production there are two tasks corresponding to each talent dimension.

In order to show the main intuitions in a simple way, without loss of generality, I assume
that there are finite types of agents. Particularly, I assume two talent levels along each talent dimension: \{H, L\}. Thus there are high-skill or low-skill entrepreneurs and high-skill or low-skill workers.

Then the possible values for each agent’s talent endowment bundle \((E, W)\) in this North economy are the following:

\[
(E, W) = \{(L, L), (H, H), (L, H), (H, L)\}
\]

Agents with endowment bundle \((L, L)\) have low level of entrepreneurial ability and also low level of worker skill; Agents with bundle \((H, H)\) are the most educated, they have both high level of entrepreneurial ability and also high level of worker skill; Agents with endowment \((H, L)\) has good entrepreneurial training but poor worker skill training; Agents with endowment \((L, H)\) have good training in worker skill but little in entrepreneurial activities.

Notice that the correlation of talent endowment along two dimensions for individual agent, \(E\) and \(W\), is one for the first two pairs of values, no matter how many there are each of them. And the correlation is negative one for the last two pairs. In general, the correlation can vary between these two extreme values. In the real world, endowment correlations between two dimensions for individual agent do differ across countries, due to different education systems.\(^{21}\)

To simplify the analysis, I define two types of North economies. The type-I North economy has talent endowment bundles consisting of the first two pairs of values listed above, with a perfect positive correlation between two dimensions of individual agent’s endowment:

\[
(E, W) = \{(L, L), (H, H)\};
\]

The type-II North economy has endowment bundles taking values of the last two pairs, with a perfect negative correlation across dimensions:

\[
(E, W) = \{(L, H), (H, L)\}.
\]

All other North economies with different endowment correlations can be generated by a linear combination of these two types of economies.\(^{22}\)

\(^{21}\)The empirical evidence for different skill endowment correlations across countries is limited in the current literature. See Ohnsorge and Trefler (2007), Krueger and Kumar (2004a) among others for preliminary evidences for the US and Europe.

\(^{22}\)Since there are four possible values of endowment bundle, which are all included in these two type of North economies. Endowment of any economy can be divided into two large groups, one containing \((L, L), (H, H)\) bundles, the other containing all \((L, H), (H, L)\) bundles. The first group is a linear combination of the type-I economies and the second is a type-II North economy combination.
When considering the case of North-South trade, I need to assume talent endowments for the South economy. A consensus seems to be that the South has caught up fast along the worker skills dimension over the past decades, while remaining relatively poor in entrepreneurial capability either because of the biased education system, poor institutions, low level of financial development or lack of entrepreneurial culture. Thus the South has a similar level of $W$ talent endowment as the North, but a relatively lower level of $E$ talent endowment. For simplicity, I assume that the South has the same level of talent endowment as the North along the $W$ dimension, and a lower level along the $E$ dimension. The potential values of talent endowment bundles $(E, W)$ for the agents in the South are then:

$$(E, W) = \{(L', L), (H', H), (L', H), (H', L)\}$$

where $H' = \theta H$ and $L' = \theta L$, $\theta < 1$. Two types of South economies with perfect positive and negative endowment correlations are then defined similarly as in the North.

### 2.1.2 Production

The technologies of production are the same in the North and South. There are two industries, service ($S$) and manufacturing ($M$), with a Cobb-Douglas production functions. I assume that in all industries talent $E$ is always more intensively used than the $W$ talent. Moreover, in the service industry, entrepreneur’s personal success is even more important. Think of the software service for example, the manager on the team performs a problem-solving task and the worker is writing and copying the codes. The output of this team largely depends on how many problems that this manager is able to solve, the worker’s contribution is only minor. Think of another manufacturing industry, automobile production, it is important for the manager to know how to organize the production process, and also very important for the worker to do a good job in actually producing the car. Thus the entrepreneurial talent is more intensively used in the service industry than in the manufacturing. The two production functions for these two industries are given by the following:

$$S = E^\alpha W^{1-\alpha}, \quad M = E^\beta W^{1-\beta}, \quad \alpha > \beta \geq 1/2; \quad (1)$$

\[23\] There are many empirical papers documenting the catch-up in manufacturing in the developing economies. In the contrast in the service sector, where there are more small business and entrepreneurial talent is more intensively used, there is no catch-up in labor productivity. See Duarte and Restuccia (2010) among others.

\[24\] In the real economy, this seems to be a reasonable assumption, given the extremely higher pay to those managerial jobs. The model does not need this assumption to generate implications on comparative advantage and gains from trade, or wage inequality; It only needs a moderate form of this assumption to generate job polarization. Detail on this will be presented in section 4.
where $\alpha$ and $\beta$ are the entrepreneurial ability ($E$) intensities in service and manufacturing industry respectively.

### 2.1.3 Preference

Agents’ welfare comes from goods consumption. Each agent’s utility from consuming two goods is given by:

$$U = C_s^\mu C_m^{1-\mu};$$

(2)

where $C_s$ is the consumption of service good and $C_m$ is the consumption of manufacturing good.

### 2.2 Random Matching

The agent matching process can be modeled in many ways. The most common candidates are the random matching process and the social planner’s optimal matching arrangement. I analyze different trade patterns with the random matching in this subsection and under optimal matching in the next.

In the random matching case, agents meet with each other in a random fashion and form teams after each meeting. Then each team chooses its task assignment, i.e. who will be the entrepreneur and who will be the worker, and at the meantime which industry to enter.

#### 2.2.1 North-North Trade

To analyze the role of multidimensional endowment in trade, I first consider two North economies with the same aggregate talent endowments along both $E$ and $W$ dimensions. The only difference is the correlation between the two dimensions within each endowment bundle. Particularly, in a Type-I North economy with a perfect positive correlation, its endowment bundles are:

$$(E, W) = \begin{cases} 
(L, L) \text{ with measure } 1 \\
(H, H) \text{ with measure } 1 
\end{cases}$$

And in a Type-II North economy with a perfect negative correlation, its talent endowments are:

$$(E, W) = \begin{cases} 
(L, H) \text{ with measure } 1 \\
(H, L) \text{ with measure } 1 
\end{cases}$$

As we can see, the value of agent measures are chosen in a way to ensure that the aggregate endowments along each dimension are the same for these two economies.
Under the random matching process, assuming agents have incomplete information and teams are formed by randomly matching two agents, in the Type-I North economy the matching outcome are the following, I use the \( \otimes \) to denote matched agents:

\[
(E,W) \otimes (E,W) = \begin{cases} 
(L,L) \otimes (L,L) \text{ with measure } 0.25 \\
(L,L) \otimes (H,H) \text{ with measure } 0.5 \\
(H,H) \otimes (H,H) \text{ with measure } 0.25 
\end{cases}
\]

After matching, each matched pair of agents have to assign the two tasks among themselves and also choose the industry (S or M) they are going to enter based on their talent endowment bundles, given the good prices and production functions. For the above matches, the effective talent bundles chosen by all matched teams in this type-I North economy will be:

\[
(E,W) = \begin{cases} 
(L,L) \text{ with measure } 0.25 \\
(H,L) \text{ with measure } 0.5 \\
(H,H) \text{ with measure } 0.25 
\end{cases}
\]

Teams with agents’ endowment bundles \((L,L) \otimes (H,H)\) will allocate the entrepreneurial task to the \((H,H)\) guy and the worker task to the \((L,L)\) agent due to the simplifying assumption that the \(E\) intensities \(\alpha\) and \(\beta\) for two industries are both greater than \(1/2\).

Analogously, in the Type-II North economy the random matching outcome are the following:

\[
(E,W) \otimes (E,W) = \begin{cases} 
(L,H) \otimes (L,H) \text{ with measure } 0.25 \\
(L,H) \otimes (H,L) \text{ with measure } 0.5 \\
(H,L) \otimes (H,L) \text{ with measure } 0.25 
\end{cases}
\]

And the effective talent bundles of for these matched teams in this type-II North economy will be:

\[
(E,W) = \begin{cases} 
(L,H) \text{ with measure } 0.25 \\
(H,H) \text{ with measure } 0.5 \\
(H,L) \text{ with measure } 0.25 
\end{cases}
\]

Notice that for these two types of North economies, for any randomly matched team, the optimal choice of effective talent bundle is unique for any good prices. It is due to the assumption on the talent intensities given the talent endowments. This assumption will be relaxed in the continuous model case.

Now I can analyze the choice of industry for each team given different good prices, together with the consumption demand for both products, I can then pin down the equilibrium prices and output quantities. Without loss of generality, I let the manufacturing good be
the numeraire and then $P_m = 1$, $P_s = P$.

In a type-I North economy, teams with effective talent bundle $(L, L)$ or $(H, H)$ will choose to enter the same industry for any given relative price $P$ since they have the same effective talent ratio. In particular, when $P = 1$ they are indifferent between producing $S$ and $M$; When $P < 1$, they choose to produce the $M$ good; When $P > 1$, they enter the $S$ industry. For teams with effective talent bundle $(H, L)$, the $S$ industry is chosen if and only if the price $P \geq \left( \frac{L}{H} \right)^{\alpha - \beta}$ (indifferent when equality holds). Otherwise, they choose to produce the $M$ good.

In a type-II North economy, teams with effective talent bundle $(H, H)$ have the same supply curve as teams with the same effective bundle in the type-I North economy. So do teams with effective bundle $(H, L)$. For teams with effective talent bundle $(L, H)$, they are more likely to choose the $M$ industry due to their high $W/E$ ratio which gives them comparative advantage in the more $W$-intensive $M$ industry. In particular, when the relative price $P < \left( \frac{H}{L} \right)^{\alpha - \beta}$, these teams will always stay in the $M$ industry.

The relative supply curves of two products for both types of North economies are shown in the Figure 1:

![Figure 1: North-North Trade](image_url)

where $A = \frac{2H^\alpha L^{1-\alpha}}{H + L}$, $B = \frac{H^\alpha L^{1-\alpha}}{H^\beta L^{1-\beta} + 2H}$, and $C = \frac{H^\alpha L^{1-\alpha} + 2H}{H^\beta L^{1-\beta}}$.

When these two types of North economies integrate to trade, the type-I North economy has a comparative advantage in the relatively more $E$-intensive industry $S$ and the type-II North economy possesses comparative advantage in the less $E$-intensive industry $M$. The intuition behind is straightforward. The high correlation between two dimensions of talent endowment in the type-I North economy means that agent with a high level of $E$ talent also has a high level of $W$ talent. However, each agent can only choose one occupation and performs the corresponding task, she has to waste her other talent. Given the same level of
aggregate talent endowment along each dimension in the two economies, a high correlation across two dimensions in the type-I North reduces its effective talent endowment that can be actually utilized. This further results in more teams with a high level talent along one dimension and a low level along the other in their effective talent bundles, generating its comparative advantage in industries with more extreme talent intensities.\textsuperscript{25}

In contrast, in a type-II North economy with a low correlation between endowment dimensions, each agent has her own specialization along certain talent dimension. Hence when two agents with different talent endowments are matched together, they will choose their occupations, i.e. task assignment, according to their specializations. No matter which industry they choose to enter, each choosing her relatively high level talent is always a dominant choice. In the end, the type-II North economy has more teams with similar (high) levels of talent along two dimensions in their effective bundles, generating its comparative advantages in industries with middle talent intensities.

I apply this intuition to the analysis of the specialization pattern between the US and continental Europe. Given that these two regions have similar level of aggregate educational resources to allocate to individuals, and there are multiple dimensions of skills, the resulted correlation for individual endowment between dimensions is higher in the US than that in Europe due to the general education system in US and the skill-specialized vocational education system in the Europe. This correlation difference alone generates comparative advantage for the US in those service industries with extreme intensities, and for the Europe in those manufacturing industries.

\subsection*{2.2.2 North-South Trade}

In the case of North-South trade integration, I also consider two types of talent endowments for the South economies. The Type-I South economy has a perfect positive correlation between the two dimensions of talent endowments:

\[
(E, W) = \begin{cases} 
(L', L) \text{ with measure } 1 \\
(H', H) \text{ with measure } 1 
\end{cases}
\]

While the Type-II South economy has a perfect negative correlation between the two talent attributes:

\[
(E, W) = \begin{cases} 
(L', H) \text{ with measure } 1 \\
(H', L) \text{ with measure } 1 
\end{cases}
\]

Under random matching, in the Type-I South economy the outcomes of agent matching

\textsuperscript{25}If there are extreme high W-intensity industries available, the type-I North will also have comparative advantage those industries.
are the following:

\[(E, W) \otimes (E, W) = \begin{cases} (L', L) \otimes (L', L) \text{ with measure } 0.25 \\ (L', L) \otimes (H', H) \text{ with measure } 0.5 \\ (H', H) \otimes (H', H) \text{ with measure } 0.25 \end{cases} \]

For the agent matches above, the effective talent bundles of all matched teams in this type-I South economy will be:

\[(E, W) = \begin{cases} (L', L) \text{ with measure } 0.25 \\ (H', L) \text{ with measure } 0.5 \\ (H', H) \text{ with measure } 0.25 \end{cases} \]

For the cross-matches where agents endowments are \((L', L) \otimes (H', H)\), as assumed \(H' = \theta H\) and \(L' = \theta L\), the talent bundle \((H', L)\) always outperforms the bundle \((L', H)\) due to the simplifying assumption that in both industries the \(E\) intensities \(\alpha\) and \(\beta\) are higher than 0.5. So the effective talent bundle will always be \((H', L)\) for these teams.

Similarly, in the Type-II South economy the possible agent matches are the following:

\[(E, W) \otimes (E, W) = \begin{cases} (L', H) \otimes (L', H) \text{ with measure } 0.25 \\ (L', H) \otimes (H', L) \text{ with measure } 0.5 \\ (H', L) \otimes (H', L) \text{ with measure } 0.25 \end{cases} \]

For the matches above, the effective talent bundles of all matched teams in this type-II South economy will be:

\[(E, W) = \begin{cases} (L', H) \text{ with measure } 0.25 \\ (H', H) \text{ with measure } 0.5 \\ (H', L) \text{ with measure } 0.25 \end{cases} \]

For those cross-matches \((L', H) \otimes (H', L)\), the effective bundle \((H', H)\) is superior to \((L', L)\) for any industry and good prices.

Since there are two types of economies both in the North and the South, there are then four different cases for North-South trade. All four possible cases are analyzed below.

**Case 1**: Type-I North with Type-I South
where $A' = A \theta^{\alpha-\beta}$.

Since $\theta < 1$ and $\alpha > \beta$, thus $A' < A$. And $H' < H$, then $(\frac{L}{H'})^{\alpha-\beta} > (\frac{L}{H})^{\alpha-\beta}$. As shown in Figure 2, the supply curve for the type-I South economy always locates above that for the type-I North economy without any intersection. Thus the price of $S$ (relative to good $M$) is always higher in the South economy. The type-I North economy always has comparative advantage in the relatively more $E$-intensive $S$ industry. In particular, as in OT, teams choose their industry and effective talent bundle accordingly. It is those North teams with effective bundle $(E,W) = (H,L)$ that stay in the $E$-intensive $S$ industry and benefit the most from trade integration with the South.

It is useful to compare our results here with the case of a standard Heckscher-Ohlin (henceforth H-O) model. The type-I North economy has exactly the same endowment of $W$ talent as the type-I South economy. The only difference is that South has a relatively lower endowment of $E$ talent, $\theta < 1$. Endowment correlations are the same. Under the H-O framework, since the relative endowment of talent $E$ in the type-I South economy is lower than that in the type-I North, the North economy has comparative advantage in the $E$-intensive industry $S$. As we can see, the same results of comparative advantages between the North and South are obtained in this model with multidimensional endowment.

**Case 2**: Type-I North with Type-II South
where $B' = B\theta^{\alpha-\beta}$, and $C' = C\theta^{\alpha-\beta}$. It can be easily verified that $B' < A < C'$ and $(\frac{1}{\theta})^{\alpha-\beta} > 1$.

Again, as shown in Figure 3, the supply curve for the type-II South economy always locates strictly above the one for the type-I North economy. The same comparative advantage and trade pattern will be obtained as in Case 1. In this case, the relative aggregate level of endowment is still the same as in Case 1. The North has a relatively higher endowment of talent $E$. In addition, the type-II South has a lower correlation of talent endowment between two dimensions. As we already know from the case of North-North trade, *ceteris paribus*, a higher correlation gives rise to comparative advantage in industries with extreme-value talent intensities. In this Case 2, the type-I North economy has a relatively higher aggregate level endowment of talent $E$, and also a higher correlation, both give it comparative advantage in the $S$ industry. Thus the traditional H-O effect and the new correlation effect work in the same direction here.

**Case 3**: Type-II North with Type-I South
where $A' < C$, the relation between $B$ and $A'$ is not clear, which depends on the values of $\theta$, $\alpha - \beta$, $H$ and $L$.

As shown in Figure 4 above, now the relative location of two supply curves is not clear. The trade pattern is also indeterminate. The intuition follows from the argument I made in case 2. In the case 3 here, on one hand the type-II North economy has a relative higher level of aggregate endowment in talent $E$, which gives it comparative advantage in the $S$ industry. On the other hand, the type-II North economy also has a lower correlation compared with the type-I South economy, which gives the type-II North economy comparative advantage in the $M$ industry with modest talent intensities. These two effects counteract each other. The equilibrium trade pattern in the integrated economy depends on the net of these two effects.

In particular, when $\theta \leq \frac{L}{H}$, we have $(\frac{L}{H})^{\alpha-\beta} > 1$ and $(\frac{1}{\theta})^{\alpha-\beta} > (\frac{H}{L})^{\alpha-\beta}$. Then it is certain that the supply curve of the type-I South economy locates strictly above that of the type-II North economy, and the North will always have the comparative advantage in the relatively more $E$ intensive $S$ industry. The intuition is that, when $\theta \leq \frac{L}{H}$, the relative level of aggregate talent $E$ endowment is sufficiently high in the type-II North economy, then the traditional H-O effect of relative factor endowment dominates the effect of correlation difference in shaping the comparative advantages between these two economies.

**Case 4: Type-II North with Type-II South**
where $B' < B$ and $C' < C$ always hold.

As shown in Figure 5, the supply curve for the type-II South economy always locates strictly above the one for the type-II North economy. Thus it is always the case that the type-II North economy has comparative advantage in the $S$ industry and the type-II South economy has comparative advantage in the $M$ industry. This case is very much similar with case 1. The correlations between two endowment dimensions in these two economies are the same. The only difference is the relative level of aggregate endowment. The type-II North has a higher aggregate endowment of talent $E$, which gives it comparative advantage in the more $E$-intensive industry $S$.

In all these cases, not only the relative aggregate endowment but also the correlation of endowment matters in trade. This leads to a key difference between this model and the standard H-O model. What determines the comparative advantages and trade patterns here is the effective endowments instead of the original gross endowment as in the H-O model. Effective endowment is defined as the part of original endowment that is actually utilized in production. It differs from the original multidimensional endowment because agent has to choose task and production is performed in teams. Thus part of each agent’s endowment bundle is unused.

The effective endowment is determined by the original gross endowment, the agent matching process and the within-team task assignments, i.e. the choice of effective bundle and industry for each team. As we have shown in the North-North case, the correlation of endowment plays an important role in shaping the effective endowment and thus comparative advantage even with the same aggregate gross endowment. In all the cases of North-South trade shown above, the comparative advantages are determined jointly by the relative original aggregate endowment and endowment correlations. Sometimes the traditional H-O
results are amplified by the effect of correlation difference, as in case 2. And sometimes the
traditional H-O results are counteracted by the effect of correlation difference, as in case 3.

2.3 Optimal Matching

In this subsection I analyze the trade patterns under optimal agent matching. A benevolent
social planner maximize the utility from goods consumption for the whole economy, given the
multidimensional endowment of each agent and the production functions in two industries,
by choosing the agent-matching arrangement, the task assignments and industry allocation
for each team simultaneously.

2.3.1 North-North Trade

First consider a type-I North economy. Since there are only two type of agents in this
economy, there are two possible matching schemes. One is self-matching, the other is
cross-matching. Under self-matching scheme, agents with the same talent endowment are
matched together. Under cross-matching scheme, agents with different talent endowments
are matched together.

The social planner chooses the optimal agent matching arrangement from the self-matching
scheme, cross-matching scheme, or a mix of two schemes. When self-matching scheme is cho-
sen, the effective talent bundles in this economy will be half \((H, H)\) and half \((L, L)\). When
cross-matching scheme is optimal, all teams have the same effective bundle \((H, L)\), again due
to the simplifying assumption that both industries are \(E\)-intensive.

Under the optimal arrangement, it is convenient to work with the production possibil-
ity frontier (henceforth PPF) for each economy. The possible PPFs for this type-I North
economy under each matching scheme are shown below in Figure 6:
Figure 6: Optimal Matching for the Type-I North Economy

The line $ab$ is the PPF under the cross-matching scheme, line $cd$ is the one under complete self-matching scheme. Line $fg$ is parallel to $ab$ and $ef$ is parallel to $cd$. Thus $efg$ is the PPF under certain mix of the two schemes. In the case shown above in Figure 6, there is no dominant matching scheme for the social planner.\footnote{There may be dominant matching scheme in other cases. For instance, when $\alpha$ and $\beta$ are very close to 0.5, it is possible that $H^\alpha L^{1-\alpha}$ and $H^\beta L^{1-\beta}$ are both less than $\frac{1}{2}(H+L)$. In this case, the self-matching scheme dominates. This is similar with assortative matching in assignment models.}

\[
H^\alpha L^{1-\alpha} > \frac{1}{2}(H + L) > H^\beta L^{1-\beta}
\]

This condition ensures that there is no dominant matching scheme for this economy.

Even without a dominant matching scheme, the social planner can still achieve any possible production point on the line $ad$ by choosing a proper mix of the two schemes. As a result, the line $ad$ is the PPF faced by the social planner in this economy.

In autarky, the social planner in this type-I North economy chooses a mix of two matching schemes to achieve a particular point on the line $ad$ to maximize the utility of consumption. Given the consumer preference on goods consumption, the equilibrium relative price of $S$
will be determined by the reciprocal of (the absolute value of) the slope for line $ad$, which is 
\[
\frac{H^\alpha L^{1-\alpha}}{\frac{1}{2}(H+L)}.
\]

Given that $H^\alpha L^{1-\alpha} > \frac{1}{2}(H+L)$ in the case of no dominant matching scheme, the absolute value of this slope is greater than one. Hence the relative price of $S$ in this economy under autarky $P = \frac{\frac{1}{4}(H+L)}{H^\alpha L^{1-\alpha}}$ is smaller than one.

When there is a dominant matching scheme, the relative price of $S$ will be $P = \frac{H^\beta L^{1-\beta}}{H^\alpha L^{1-\alpha}} < 1$ when cross-matching dominates; and 1 when self-matching dominates.

In a type-II North economy, the optimal matching scheme is always the cross-matching scheme. This is because two teams with effective talent endowment $(H, H)$ will always outperform two teams with effective endowments $(H, L)$ and $(L, H)$ in any industry. Hence the PPF faced by the social planner in a type-II North economy is always the one under complete cross-matching, a negative 45 degree line. Thus the relative price of $S$ always equals one under autarky.

Notice that in all the cases, the relative good price of $S$ in type-I North is always less or equal to one under autarky, with equality holds only when the self-matching scheme dominates, i.e. when $H^\alpha L^{1-\alpha} < \frac{1}{2}(H+L)$. In most cases, the type-I North economy has a lower price for good $S$ than the type-II North. Thus the type-I North has comparative advantage in the $S$ industry. Only when the self-matching scheme also dominates in the type-I North economy, the two prices for good $S$ in two economies will equal. Only in that case, there is no incentive to trade between these two economies.

In general, when these two types of North economies open to trade, the type-I North economy has comparative advantage in the industry $S$ with a more extreme talent intensity. And the type-II North economy has comparative advantage in the $M$ industry with a modest talent intensity. Analogously, when there is another industry with very high $W/E$ intensity, the type-I North economy will also possess the comparative advantage in that industry. A more general theory for this mechanism shaping the trade patterns between these two types of North economies when there is a continuum of industries is presented in the section 3.

### 2.3.2 North-South Trade

Again, I first look at a type-I South economy. The social planner also chooses the agent matching scheme from self-matching, cross-matching, or a mix of the two schemes. Under the self-matching scheme, agents with the same talent endowment are matched together. Hence half of the teams have effective bundle $(H', H)$ and the other half have effective bundle $(L', L)$. Under cross-matching, agents with different talent endowment are matched and all teams choose the same effective bundle $(H', L)$, due to the assumption that the $E$ intensities for both goods are greater than 1/2. The PPFs for each matching scheme are
shown in the Figure 7 below.

The line $ab$ is the PPF under complete cross-matching scheme; The line $cd$ is the PPF under the self-matching scheme; And the $efg$ is the PPF under certain mix of the two matching schemes. In the case shown in Figure 7, $H^\alpha L^{1-\alpha} > \frac{1}{2} \theta^\alpha (L + H)$ and $\frac{1}{2} \theta^\beta (L + H) > H^\beta L^{1-\beta}$, there is no dominant agent matching scheme. Again, as in the type-I North economy, the social planner here can achieve any outcome along the line $ad$ by choosing proper mixes of two matching schemes, hence the thick line $ad$ is the PPF faced by the social planner.

Notice that the conditions for no dominant matching scheme in this type-I South economy can be simplified into:

$$H^\alpha L^{1-\alpha} > \frac{1}{2} (L + H) \text{ and } \frac{1}{2} (L + H) > H^\beta L^{1-\beta}$$

which is the same as the conditions for no dominant matching scheme in the type-I North economy.

Consider the trade between the type-I North and type-I South economies, for the type-
I South economy, when there is no dominant agent matching scheme, the social planner chooses a best mix of two schemes to achieve the equilibrium production point (also autarky consumption point) at $f$. The relative price for good $S$ is $P^* = \frac{\theta^\beta \frac{1}{2}(L+H)}{\theta^\alpha H^\alpha L^{1-\alpha}} = \frac{\theta^\beta}{\theta^\alpha} P$. Since $\theta^\beta > \theta^\alpha$, this price\textsuperscript{27} $P^*$ is greater than that of a type-I North economy $P$. North has comparative advantage in industry $S$.

When $\frac{1}{2}(L+H) > H^\alpha L^{1-\alpha}$, the self-matching scheme dominates in both economies. The autarkic price for $S$ equals one in the North, the price for $S$ in the South is $\frac{\theta^\beta}{\theta^\alpha}$, which is greater than one. In this case, South still possesses comparative advantage in the $M$ industry. When $H^\beta L^{1-\beta} > \frac{1}{2}(L + H)$, the cross-matching scheme dominates in both economies. The autarkic price for $S$ in the South is $\frac{H^\beta L^{1-\beta}}{H^\alpha L^{1-\alpha}} = \frac{\theta^\beta}{\theta^\alpha} H^\beta L^{1-\beta}$. Since $\frac{\theta^\beta}{\theta^\alpha} > 1$, the South still possesses the comparative advantage in industry $M$.

Now consider the trade between the type-I South and type-II North economies, the relative price for good $S$ is always 1 in the type-II North economy. In cases when there is no dominant matching scheme or the cross-matching scheme dominates in the type-I South economy, the relative price for good $S$ in the South may be greater or less than one. Only when the self-matching scheme dominates in the South, the relative price of good $S$ is certain to be greater than one. Thus only in this case, the type-I South is sure to have comparative advantage in the $M$ industry. This result is very similar as that we get under the random matching process. The trade pattern between a type-I South and type-II North may be undetermined with all informations we have here.

Finally, I consider the trade between a type-II South economy and the North economies. In a type-II South economy, the optimal matching scheme is always cross-matching, as two teams with effective talent endowment $(H', H)$ always outperform two teams with effective endowments $(L', H)$ and $(H', L)$ no matter which industry they choose. Hence under autarky, the relative good price for good $S$ is always equal to $\frac{\theta^\beta}{\theta^\alpha}$, which is always greater than one. Hence in the case of integration with North economies, either type-I or type-II North economy, this type-II South economy always has comparative advantage in the more $W$ intensive $M$ industry.

From the analysis in the previous and current subsection, we can see that very similar results are generated under the random agent matching process and the optimal agent matching chosen by a benevolent social planner. These results have universal applications.

\textsuperscript{27}I use asterisk to denotes variables in the South whenever needed.
2.4 Gains From Trade in the Baseline Model

I analyse the gains from trade in this baseline model in this subsection. I compare a new source of gains from trade with the conventional gains that identified in the literature. In summary, under random matching, the two economies with different comparative advantages will gain from trade in a conventional way. When assumptions on endowment and production change, the new source of gains from trade may occur. Under optimal matching, the new source of gains from trade may still exist even under the assumptions of this baseline model.

2.4.1 Gains from Trade in Random Matching

Under random matching, the PPF of each economy is fixed given all the assumptions in the baseline model. This is because the share of teams are fixed, and for each team the choice of effective endowment bundle is also fixed. Thus in this baseline model, the gains from trade is the same as those already identified in the literature.

There are overall gains from trade integration in this baseline open economy model under random matching. However, the distribution of these gains across countries might be uneven. In this simple baseline model, with all the assumptions on endowment and industry intensities, we have the following lemma on gains from trade.

**Lemma 1.** With random matching, the economies whose relative good price changes from autarky to integration gain from trade. The economies with the same price after integration stay the same in welfare level.

**Proof.** Without change of production technology, the production possibility frontier of each economy is not changed due to the same matching outcome and same effective talent bundle choices under random matching. This is because given the assumptions on production functions and endowment levels, the effective bundle chosen by each team is already a dominant choice for every team and for all relative good prices. An economy will gain if the new price differs from its autarky price by the revealed preference. And due to the same reason, welfare stays the same if price does not change from autarky to integration for some other economies.

When there is a continuum of talent levels and industries, this lemma may be gone. Then there might be changes of the effective endowment, i.e. the production possibility frontier, due to the possible change of effective endowment bundle choice for each team. In that case, there exists another possible source of gains from trade generated by the option to switch effective talent bundle within each team. This new source of gains from trade differs from the conventional gains in the sense that the effective endowment changes.
2.4.2 Conventional Gains and New Gains

In the standard trade models, such as Ricardian model and H-O model, the production and trade patterns in the open economy are determined by the endowments and technology. Each economy utilize all of its endowments to achieve the highest consumption (welfare) under autarky. The difference in the autarkic good prices across countries provides an incentive to trade. Each country will gain from the trade integration as consumers can consume goods at a lower price index and total output increases because more resources are allocated into its comparative advantage industries. This form of conventional gains from trade still exists in my model. However, in this framework, it is the effective endowment instead of the initial endowment that determines the production and trade patterns. These conventional gains from trade come from the change of prices and reallocation of the utilized endowments. In my model, a new source of gains originates from the change in the effective endowment itself.

As previously shown in this paper, the effective endowment is determined by the initial endowment, the agent-matching scheme and the task-assignment on each team. When open to trade, because of the price changes, the social planner may choose to change the agent matching outcome or switch task assignment within teams to improve the overall usage of initial endowment. For example, think of a small type-I North economy with no dominating matching scheme under autarky, the relative price of $S$ is $P = \frac{1}{H^{\alpha}L^{1-\alpha}}\frac{H+L}{H^{\alpha}L^{1-\alpha}}$, which is less than one in this case. When it opens up to trade with the rest of the world, suppose the world price for good $S$ is greater than $P$, then this small type-I North economy will completely specialize in producing good $S$. It will change the mix matching scheme into complete cross-matching. In this way, this economy improves its usage to its initial endowment. Notice that, when there is dominant agent matching scheme, then effective endowment can not be achieved by changing agent matching outcome when open to trade.

To show how the effective endowment can be improved by changing within team task assignment, I need to assume another industry $A$ with low (less than $1/2$) $E$ intensity but high $W$ intensity. Again consider a small type-I North economy, suppose now the cross-matching scheme is dominant, then all teams are formed by two agents with different endowment bundles. Then the effective bundle chosen by teams operating in the $S$ industry and $M$ industry is $(H, L)$, and the effective bundle chosen by teams operating in the $A$ industry is $(L, H)$. Suppose the world prices for good $M$ and $A$ are both lower than autarkic prices in this small type-I North economy, and price for good $S$ is higher than the autarkic price in this economy, when open to trade this economy will shift all its resources (teams) into the $S$ industry. This reallocation of existing teams is an conventional way to gain from trade. Since cross-matching is a dominant matching scheme, it can not improve effective endowment through agent re-matching. Instead what it can do is changing the effective talent bundles
for those teams moving from industry $A$ to industry $S$, that is changing all those effective bundles $(L, H)$ into $(H, L)$ through task re-assignment within those teams. Through this change, the final output of good $S$ is improved.

The following part of the paper presents two examples of this new gains from trade. The existence of this new source of gains from trade in a general model is presented in section 3.

### 2.4.3 The New Gains From Trade: Examples

The new gains from trade in my model originates from the change of the effective endowments, either through re-matching the agents or re-assigning the tasks within teams. To clarify, I present two examples for each of the two cases.

Let $\alpha > \frac{1}{2} > \beta$, and suppose in the type-I North economy the cross-matching scheme is dominant. Then the PPF for this economy is shown in Figure 8. Thus the teams in this economy have the same effective talent bundle choices, $(H, L)$ or $(L, H)$. For those teams entering the $S$ industry the bundle $(H, L)$ is preferred, and for those producing $M$ the bundle $(L, H)$ is chosen. The equilibrium allocation of teams determines the equilibrium effective endowment in this economy.

As shown in Figure 8, $E$ is the equilibrium under autarky. When open to trade, suppose that the world price of good $M$ is higher ($P'_S < P_S$), then this economy will completely specialize in the $M$ industry. If the teams can not switch the effective talent bundle by changing the task assignment within teams, then the equilibrium in the open economy will be $E''$ and consumption is $C''$. However, if the teams moving from $S$ industry to the $M$ industry are able to switch their effective bundles by re-assigning the tasks within teams, the open economy equilibrium will be $E'$ and consumption $C'$. The consumption at $C'$ is
strictly better than that at $C''$. Thus this economy gains from the switching of effective talent bundles within teams.

In the optimal matching process for a type-I North economy, when there is no dominant matching scheme, the social planner chooses a proper mix of the two matching schemes to maximize the welfare of the economy overall. The PPF for the social planner is the line $ad$. In contrast, if the social planner is not able to change the agent matching outcome after opening to trade, the PPF faced by her will be an inferior set below $ad$. Hence, allowing possible agent re-matching introduces potential gains from trade.

Preceding are examples of gains from trade through agent re-matching and within team task re-assignment. I now present an example of another possible gains from trade in my framework, which is very similar with those in scale economies.

Consider again a type-I North economy, under the optimal matching, the PPF facing the social planner is $ad$ as shown in Figure 9 for the case when there is no dominant matching scheme. And $f$ is the optimal production point chosen by the planner under autarky.

When open to trade, $efg$ will be the PPF in the open economy for this country if the cost of breaking up existing teams and forming new teams is infinite. Suppose now the planner is able to create a market platform for those broken up agents to form new teams with a fixed cost. Then the total cost of breaking up existing teams and forming new teams features decreasing return to scale. The more teams a planner chooses to break up, the less the average cost. Then the PPF facing this social planner consists of two curves starting from the autarky point $f$, which is $e'fg'$ in Figure 9:
Notice that, in this case, the production possibility set for this country after opening to trade is a non-convex set.

The non-convexity of the production possibility set is similar with the one in scale economy models. Thus this potential new source of gains from trade also resembles that in scale economy models. However, the mechanism behind the gains here is totally different from that in scale economy models. Again this extra source of gains from trade comes from the potential changes of the utilized effective endowments in this economy. In contrast, the gains in scale economies come from the increasing return to scale in production.

I summarize the differences and parallels between this framework and scale economies in generating a non-convex production set, hence the extra gains from trade. First, production is performed in teams here, in contrast individuals produce in the scale economy model. Second, the production technology features constant returns to scale in this model while increasing returns to scale in the scale economy model. Third, agents on each team perform asymmetric tasks here. The extra gains from trade disappears if tasks on the teams are symmetric, as in Grossman and Maggi (2000). Fourth, the talent endowment for agent is multi-dimensional and each agent has to trade-off between utilizing different dimensions of their talents. Thus the effective endowment that are actually being used is not the initial endowment. In contrast, in the scale economies, there is no occupational choice and the
2.5 Wages and Job Polarization

In this subsection, I turn to another important question in the open economy, which is the effect of trade on the wages and employment.

The key intermediate step to investigate this problem is to determine the surplus division within teams.

2.5.1 Wage Determination

In the random matching case without re-matching after separation, each matched team will enter into production and divide the output evenly within teams. This is because when they bargain on their shares of surplus, they both have the same zero payoff in case of separation as the threat point. When there is search, then separated agents can enter the matching market and form new teams again. In this case, the threat points of each agent depends on the expected value of her endowment bundle.

In the type-I North economy, agent with endowment $(H, H)$ has probability 0.5 to be matched with a $(H, H)$ agent and 0.5 to a $(L, L)$ agent. Suppose that she has a bargaining power of $\gamma$ when she is matched with $(L, L)$, thus her wage is $\gamma H^\alpha L^{1-\alpha}$ and agent $(L, L)$'s wage is $(1 - \gamma) H^\alpha L^{1-\alpha}$. When $(H, H)$ is matched with a same type agent, her payoff is simply $0.5H$; when $(L, L)$ is matched with a same type agent, his payoff is $0.5L$.\(^{28}\)

Assume a extreme case that search cost is zero, then the value of the separation option is $0.5\gamma H^\alpha L^{1-\alpha} + 0.25H$ for agent $(H, H)$ and $0.5(1 - \gamma) H^\alpha L^{1-\alpha} + 0.25L$ for agent $(L, L)$. The efficient bargaining solves the following problem by choosing $\gamma$:

$$\argmax \left( \gamma H^\alpha L^{1-\alpha} - (0.5\gamma H^\alpha L^{1-\alpha} + 0.25H) \right) \left( (1 - \gamma) H^\alpha L^{1-\alpha} - (0.5(1 - \gamma) H^\alpha L^{1-\alpha} + 0.25L) \right)$$

Solving this problem gives the efficient bargaining outcome for agent $(H, H)$:

$$\gamma_1 = \frac{1}{2} + \frac{H - L}{4H^\alpha L^{1-\alpha}}$$

Similarly, in the type-II North economy, when agent $(H, L)$ is matched with a $(L, H)$ agent, she gets $\gamma H$ and her partner gets $(1 - \gamma)H$. When she is matched with a same type agent, they both get $0.5H^\alpha L^{1-\alpha}$; when two $(L, H)$ agents are matched, they each get

\(^{28}\)For simplicity, I neglect the good prices in the autarky analysis, assuming relative price equals one. This is a convenient short-cut in comparing the autarkic wage inequality levels for the two types of North economies.
0.5L^βH^{1-β}. The efficient bargaining solves the following problem by choosing $\gamma$:

$$\arg\max (γH - (0.5γH + 0.25H^αL^{1-α})) \left( (1 - γ)H - (0.5(1 - γ)H + 0.25L^βH^{1-β}) \right)$$

Solving this problem gives the efficient bargaining outcome for agent $(H, L)$:

$$\gamma_2 = \frac{1}{2} + \frac{H^αL^{1-α} - L^βH^{1-β}}{4H}$$

Following the assumptions in the baseline model, it is obvious that:

$$H > H^αL^{1-α} > L^βH^{1-β} > L.$$ 

Thus the wage inequality between two types of agents is higher in the type-I North economy than in the type-II North economy. The intuition is that in the type-II North economy, low correlation means for every agent has his own specialization, no one is significantly dominated by the other in all dimensions. Thus the bargaining power allocation within teams is closer to equalization because of the lower endowment correlation. In contrast, in the type-I North economy, agent with endowment $(H, H)$ outperforms agent $(L, L)$ in both entrepreneurial ability and working skills dimensions. As a result, there is higher inequality level in the type-I North economy than in the type-II North economy in equilibrium due to different correlations.

In a competitive environment under optimal matching, since there are many teams of the same type in a particular industry, each agent on the team gets the share that she contributes to the total team surplus. This is because there many teams with the same agent endowments. There is no searching cost, each team can find many alternatives for any one of its member. In this case each agent’s wage is determined by the industry intensity and the total team surplus.

For the type-I North economy, I consider different optimal matching arrangements for the social planner. If pure self-matching dominates, then relative price of $S$ relative to $M$ is 1. Agent $(H, H)$ gets a wage $\frac{1}{2}H$, and agent $(L, L)$ gets $\frac{1}{2}L$.

If pure cross-matching dominates, relative price for $S$ will be $P_s = \frac{H^βL^{1-β}}{H^αL^{1-α}}$. In the $S$ industry, $(H, H)$ type agents get a wage $αH^βL^{1-β}$, and $(L, L)$ agents get $(1 - α)H^βL^{1-β}$. In the $M$ industry, $βH^βL^{1-β}$ is the wage for $(H, H)$ agents and $(1 - β)H^βL^{1-β}$ for $(L, L)$ agents.

If the optimal matching is a mix of the two, the relative good price is $P_s = \frac{1}{2}\frac{(H+L)}{H^αL^{1-α}}$. In the $S$ industry, $(H, H)$ agents get $α\frac{1}{2}(H+L)$ and $(L, L)$ agents get $(1 - α)\frac{1}{2}(H+L)$. In the $M$ industry, $(H, H)$ agents get $\frac{1}{2}H$ and $(L, L)$ agents get $\frac{1}{2}L$.

For the type-II North economy, cross-matching always dominates. In the $S$ industry,
wage for \((H,L)\) agents is \(\alpha H\), \((1 - \alpha)H\) for \((L,H)\) agents; In the \(M\) industry, \((H,L)\) agents get \(\beta H\) and \((L,H)\) agents get wage \((1 - \beta)H\).

From these wages, we see that the wage inequality is higher in the \(S\) industry. The high-correlation type-I North thus also have higher wage inequality than the type-II North under optimal matching.

As we can see from the analysis in this subsection, the case of random matching with search and the competitive case yield very similar results on wages. For simplicity, when I analyse the wage and job evolutions I assume the wage determination in the competitive environment.

In order to analyse the job polarization, we need to first identify what jobs are top-wage jobs, middle-wage jobs and bottom-wage jobs. In the high-correlation type-I North economy, in most cases,\(^{29}\) we have that entrepreneurs in \(S\) industry earn the highest wage and workers in \(S\) industry earn the lowest wage. On the other hand, entrepreneurs and workers in the \(M\) industry earn middle-level wages. For the low-correlation type-II North economy, we always have that entrepreneurs in \(S\) industry earn the highest wage and workers in \(S\) industry earn the lowest wage; Entrepreneurs and workers in the \(M\) industry earn middle-level wages.

2.5.2 Trade and Wage Inequality

In the case of North-North trade, the type-I North has comparative advantage in the \(S\) industry, those teams with endowment \((H,H) \otimes (L,L)\) benefit the most from this integration. They are the top-wage entrepreneurs and bottom-wage workers in this industry. The inequality in type-I North will then increase. The opposite is true for the type-II North economy. This is true when we apply this into the comparison between the US and the EU. The US has a higher inequality level than the EU because the endowment correlation shaped by the general education system in the US is higher than that in the EU shaped by the skill-specialized vocational education system in those European countries.

The more interesting case to consider is the North-South trade. From the previous analysis we know that the South economy has comparative advantage in the relatively more \(W\)-intensive \(M\) industry. Two types of North economies both have comparative advantage in the \(E\)-intensive \(S\) industry. More teams in the North will choose to move into the \(S\) industry. Since in either type of North economy, \(S\) industry has the higher wage inequality than the other industry, inequality levels increase in both economies when open to trade with the South.

\(^{29}\)In all cases if \(\frac{H}{L} < \frac{\alpha}{1 - \alpha}\).
2.5.3 Globalization and Job Polarization

Over the last three decades or so, most of the North economies has experienced a pattern of job evolution, the “job polarization”. It denotes the fact that the employment growth for low-wage and high-wage jobs has increased more than those middle-wage jobs. This phenomenon has been addressed pretty well in the closed economy with the task approach by David Autor and other labor economists. Here I show that this simple baseline model may also generate such a job change pattern in the open economy, specifically in the North-South integration.

As shown in the wage determination subsection, in both types of North economies, entrepreneurs in the $S$ industry earn the highest wages in the economy and workers in the $S$ industry earn the lowest wages. Those entrepreneurs and workers in the $M$ industry earn middle-level wages.

After integration with the South in the globalization era, those agents working in the $S$ industry in the North benefit the most due to the relative price increase for $S$. They are the top-wage entrepreneurs and bottom-wage workers in the North. After integration there will be more such matches formed and entering the $S$ industry in the North. As a result, the employment of these top-wage jobs and bottom-wage jobs increases relative to middle-wage jobs in the $M$ industry. Thus a pattern of “job polarization” exist in all the North economies.

Notice that even though this “job polarization” exists in all North economies. The extent of this pattern may differ in different types of North economies. Additionally, the effects on wage evolution will definitely be different for different types of North economies. In fact, there are indeed evidences for divergent wage changes in different North economies. Autor et al (2006) documents both “job polarization” and “wage polarization” in the U.S. However, the second polarization pattern is not observed in the European countries.

All in all, this baseline model provide an alternative explanation of the job polarization using North-South trade, highlighting the potential role for globalization behind these observed labor market outcomes in the North economies.

3 The Continuous Model

In this section I present a more general model with a continuum of industries and continuous multidimensional talent endowment.

There are a continuum of industries with $E$ intensities range from $\alpha$ up to $\bar{\alpha}$. For any particular industry with intensity $\alpha_n$, $n \in [0, 1]$, the production function is given by:

$$Y_n = E^{\alpha_n} W^{1-\alpha_n}.$$  (3)
The $n$ industries are ranked such that for $n > n'$, $\alpha_n > \alpha_{n'}$.\footnote{A more general form of constant return to scale production function with asymmetric tasks is $Y = \left[ \alpha E^{\sigma-1} + (1 - \alpha)W^{\sigma-1} \right]^{\frac{1}{\sigma-1}}$. I use the Cobb-Douglas form for simplicity, all the results are the same.}

The talent endowments of agents in this economy has two dimensions: $(E,W)$. Each follows a distribution with probability density function $g(E)$ and $h(W)$ respectively, $E \in [\omega, \bar{\omega}], W \in [\omega, \bar{\omega}]$. The correlation between the two endowment dimensions is $\rho$, $\rho \in [-1, 1]$.\footnote{A more general form of constant return to scale production function with asymmetric tasks is $Y = \left[ \alpha E^{\sigma-1} + (1 - \alpha)W^{\sigma-1} \right]^{\frac{1}{\sigma-1}}$. I use the Cobb-Douglas form for simplicity, all the results are the same.}

### 3.1 Agent Matching

I consider two different agent matching assumptions, the random matching and the social optimal matching. Under random matching, agents have incomplete information about others’ endowment before matching. The matched agents observe each other’s endowment bundle and then choose task assignment, i.e. the effective bundle, and industry to maximize team output. The wages within each team are determined by solving the bargaining problem. Each agent’s bargaining power depends on his alternative option value, which is the expected value of his endowment bundle.

Under the optimal matching mechanism, the benevolent social planner has complete information about agents' endowments. He chooses the agent matching arrangement, the task assignments and industry choices for each team in order to maximize the aggregate social welfare, given the available industries. In this competitive environment, each agent’s wage equals her share of contribution to the final output. Particularly, in the Cobb-Douglas production case these shares of contribution to the final output equal the talent intensities.

### 3.2 Generalized Trade Theories

Under this new framework I reinvestigate several well-known trade theories. Generalizations of some established trade theories are found, while some other trade theories no longer exist under new settings. Also new implications on trade and wages are drawn from this new model.

The framework in this paper shares many common features with the OT paper. They investigate how agents with different talent ratios choose their industries. This model mimic their logic after teams are formed and effective bundles are chosen. When teams with effective talent bundle of different ratios choose their industries, it follows exactly the same rationale as the agents in OT model. So the key of my proof in this paper lies in the determination of effective talent bundles.
3.2.1 The Generalized Heckscher-Ohlin Theorem

Compare two economies, North and South, with different relative talent endowment across two dimensions. Assume the two economies have the same endowment distribution along the \( W \) dimension. The talent endowment along the \( E \) dimension is lower in the South. Other aspects of endowment distributions are all the same, such as the correlations, available industries and technologies. Particularly, for any agent in the North with endowment \((E,W)_N = (E_i,W_i)\); in the South there is an agent with endowment \((E,W)_S = (\theta E_i,W_i)\), where \( \theta < 1 \).

**Proposition 1.** Given that South has relative less endowment along the \( E \) skill dimension, ceteris paribus, the North always has comparative advantage in those industries with high \( E \) intensities.

**Proof.** Since the only difference between two economies is that agents in the South have lower levels of \( E \) endowment. With random matching process, the agent matching outcome in two economies will be the same in the sense that for each matched team in the North with endowments \((E_i,W_i) \otimes (E_j,W_j)\), there is a counterpart team in the South with endowments \((\theta E_i,W_i) \otimes (\theta E_j,W_j)\). When producing a particular good with \( E \) intensity \( \alpha \), these two teams will choose effective talent bundle in the same way because

\[
(\theta E_i)^\alpha W_j^{1-\alpha} \geq (\theta E_j)^\alpha W_i^{1-\alpha} \iff E_i^\alpha W_j^{1-\alpha} \geq E_j^\alpha W_i^{1-\alpha}.
\]

Thus in the production possibility set along the PPF, the maximum output potential of any good in the North will be higher than that in the South by \( (\frac{1}{\theta})^\alpha \) times. Since \( \frac{1}{\theta} > 1 \), the higher \( E \) intensity \( \alpha \) is, the more increase in output potential in the North than that in the South. As a result, the North has comparative advantage in those industries with high \( E \) intensities.

Under the optimal matching, consider any pair of goods with \( E \) intensities \( \alpha_i \) and \( \alpha_j \), \( \alpha_i > \alpha_j \). Given the optimal output of a good in the South, the North can always increase the output potential of that good by at least \( (\frac{1}{\theta})^\alpha \) times through the increase of the talent \( E \) level, without changing the agent matching outcome or task assignments. Since North has a higher level endowment of \( E \) talent, the optimal matching and task assignment there allow it to utilize more of its \( E \) talent. More high level \( E \) talent endowment will be actually utilized. In this case, the higher the \( E \) intensity of an industry, the more output of it can be improved in the North from the optimal matching in the South. Hence again the relative output of the two industries \( \alpha_i \) and \( \alpha_j \) is always higher in the North than the South. This ratio is even higher under the optimal matching than that under the random matching process. Thus the
North economy always has a PPF more in favor of those more $E$ intensive goods than the South.

In the standard H-O theorem, a country will export goods that use its abundant factors intensively, and import goods that use its scarce factors intensively. As shown by Proposition 1, this H-O theorem still holds with multidimensional endowment.

**Proposition 2.** Assume the same $W$ endowment in both economies and less $E$ in the South, ceteris paribus, the effective endowment along the $W$ dimension in the South is always higher or equivalent to that in the North.

*Proof.* This lemma directly follows from proposition 1. Under random matching, the two economies have the same effective endowment along the $W$ dimension, since for each pair of randomly matched teams in two economies their choice of effective bundles are the same, as shown in the proof of proposition 1. Under optimal matching, the North economy is more willing to sacrifice some $W$ talent to utilize the higher level $E$ talent than the South. As the North and South have the same endowment distribution along the $W$ dimension, in the optimal matching equilibrium the South ends up with a relatively higher effective endowment of $W$ talent than the North.

This proposition is to emphasize the importance of effective endowment. In this new framework with multidimensional endowment, it is the effective endowment that determines the trade patterns. And for two economies with the same aggregate original endowment, their effective endowments that are actually utilized in production may differ. Thus it is important to investigate how the effective endowment is generated in different economies. This paper takes a first step in this direction.

### 3.2.2 Factor Price Equalization

In the H-O model, the relative prices for two identical factors of production in the same market will eventually equal each other because of competition.

However in this model, the factor prices are usually not equalized, since the return to a particular factor also depends on the factor matched with it. In the multi-dimensional endowment case, the return to a particular dimension talent also depends on the endowments along other dimensions, since they will affect the final choice of effective talent bundle. One particular dimension of talent may end up idle without any price.

Two agents from one country with the same talent endowment along all dimensions tend to have the same (at least expected) return. However, two agents with the same talent
endowment in countries with different endowments may differ in their expected returns, due
to the different pools of potential team mates they are able to find.

3.2.3 The Generalized Stolper-Samuelson Theorem

In the H-O model, a rise in the relative price of a good will lead to a rise in the return to
that factor which is used most intensively in the production of the good, and conversely, to
a fall in the return to the other factor.

Similarly to OT, in this model the industry choice is governed by the relative talent ratio
in each team’s effective talent bundle. Each matched team chooses her industry according
to her effective talent ratio. When the price of a particular good \( \alpha_i \) increases, the returns
to those effective bundles with this talent ratio \( \frac{\alpha_i}{1-\alpha_i} \) are increased. Since factor prices are
not equalized in this framework, we can not tell as in the standard H-O model whether the
return to certain dimension of talent increases or not.

3.2.4 The Generalized Rybczynski Theorem

In standard H-O model, when only one of two factors of production is increased there is a
relative increase in the production of the good using more of that factor. This leads to a
corresponding decline in the relative price of that good as well as a decline in the production
of the good that uses the other factor more intensively.

Within the current framework, a similar result holds. A universal increase across all
agents in the endowment level of certain dimension increases the production of goods with
high intensities in that talent. This is because after the increase there are more effective
bundles with high ratios of that talent. Similar with OT, this means there are more teams
choosing to produce goods with high intensities in that talent. On the other hand, the
production of goods with small intensities in that talent shrinks.

**Theorem 1.** In the framework with multidimensional endowment and team production, an
increase in one dimension of talent endowment increases the relative production of goods
using that dimension more intensively and decreases the relative production of goods using
the other dimensions more intensively.

**Proof.** I illustrate this theorem by doing comparative statics analysis between two equilibri-
ums before and after the increase of talent endowment. Given equilibrium of the economy,
each team has an effective talent bundle \((E,W)\) utilized in production of certain industry
\( \alpha \). Now consider an universal increase in the endowment levels of one particular talent di-
mension, without loss of generality, assume that the \( E \) talent levels are all increased by \( \lambda \)
times. And \( \gamma \) is only slightly greater than 1, in order to keep all the other characteristics of endowment distribution unchanged, such as the variance, correlation etc.

Under random matching, after the talent increase the agent-matching outcome is not changed, but for each team the two choices of effective bundle both become more \( E \)-intensive. If the effective bundle choices, hence industry choices, of all teams are still the same as before the increase, then the output of industry \( \alpha \) increases by \( \lambda^\alpha \) times, where \( \alpha \) is the \( E \) intensity of this industry. In this case, the output of industries with high \( \alpha \) increases more than that of industries with low \( \alpha \) values. On the other hand, if some teams choose to switch their effective bundles and hence industry choices, it can only be the case that they are moving from less \( E \)-intensive industries into more \( E \)-intensive industries. This argument can be easily shown by contradiction. If a team moves from a more \( E \)-intensive industry to a less \( E \)-intensive industry after the increase of talent \( E \), then they will choose that less \( E \)-intensive industry before the increase. The reason is similar with the proof of proposition 1. Hence under random matching, the Rybczynski theorem extends into this new framework.

Under optimal matching, the proof of this theorem works with the new PPF for this economy after the increase of talent \( E \). For any pair of two goods, the increase of talent \( E \) to \( \lambda E \) increases the output possibility of all goods. And the more \( E \)-intensive a good is, the higher the increase of its output potential will be. In the end, the new PPF is more biased to those more \( E \)-intensive goods. Then the new optimal choice of output composition consists of more \( E \)-intensive goods and less \( W \)-intensive goods relative to the optimal choice before the increase. Thus the Rybczynski theorem still holds under optimal matching in the new framework.

OT also proved a generalized version of Rybczynski theorem, which is very similar with the one shown above. There the increase of the correlation between \( s \), the talent ratio, and \( l \), the absolute level of the ratio, increases the relative level of high-\( s \) talent bundles, hence the relative production of those goods with high-ratio intensities. Here in my framework, the increase in the level of one talent increases the ratios of that talent for all endowment bundles in this economy, resulting in more high-ratio effective bundles in that talent and thus more output of goods with high-intensity in that talent. In fact, the Theorem 4 in OT on the effect of an increase in the distribution mean of \( s \), is very similar in effect as the mechanism presented here in Theorem 1.

3.2.5 Correlation and Trade

There is not many measures in the literature for the correlation between endowment dimensions. But this difference do exist due to different education systems in different countries.
For instance, the most recognized vocational education system in EU provide the economy with many specialized workers with some specific skills. They are proficient in performing certain tasks but poor in others. While in US education system where the general purpose technology is more emphasized provide students with general trainings in all dimensions. The difference in skills between agents in US lies more in absolute levels of skills instead of comparative differences.

The role of correlation between endowment dimensions in shaping the wage inequality has been emphasized by Gould (2002). He shows that workers in the US are increasingly finding that they are either good in all sectors or bad in all sectors. The negative effect of comparative advantage on inequality from the Roy model is decreasing in the US economy. The level of inequality is rising in US as the economy is increasingly characterized by the pursuit of absolute advantage rather than comparative advantage.

The International Adult Literacy Survey (IALS) has shown that people in different countries do have different mix of different talents/skills. The correlations do differ across countries with different education systems.

In this part, I will investigate the role of this endowment correlation in shaping the effective endowment, hence the production, comparative advantage, and trade patterns.\(^{31}\)

**Theorem 2.** Consider a world with two economies that are identical except for the correlation between endowment dimensions \(E\) and \(W\). (1) There exists an equilibrium. (2) In equilibrium there are two cut-off industries \(\alpha_i\) and \(\alpha_i\) such that the high-\(\rho\) economy imports all goods with middle \(E/W\) intensities \((\alpha_i < \alpha < \alpha_i)\) and exports all goods with extreme \(E/W\) intensities \((\alpha < \alpha_i\) or \(\alpha > \alpha_i)\).

**Proof.** Given the Theorem 3 in OT, I only need to prove that in this model the effective bundles in the high-\(\rho\) country are more unequally distributed.

I show this by assuming a specific distribution for both dimensions of the talent endowment, which is the normal distribution. Particularly the talent endowment of \(E\) and \(W\) follow the following bivariate normal distribution:

\[
\begin{bmatrix} E \\ W \end{bmatrix} \sim N \left( \begin{bmatrix} \mu \\ \mu \end{bmatrix}, \begin{bmatrix} \sigma_E^2 & \rho \sigma_E \sigma_W \\ \rho \sigma_E \sigma_W & \sigma_W^2 \end{bmatrix} \right),
\]

where \(\rho\) is the correlation between \(E\) and \(W\). When considering the role of correlation, I set \(\sigma_E = \sigma_W\) to restrict out the role of relative diversity, which will be analysed later.

\(^{31}\)Notice the difference between the correlation here and the one in OT. The correlation here is between different endowment dimensions for each agent. In OT instead, each agent’s endowment is characterized by his talent ratio and the talent level. The correlation in OT is the correlation between this ratio and level for each agent.
Under normality the expectation of $E$ given $W$ is

$$\xi(E|W) = \mu + \rho \frac{\sigma_E}{\sigma_W} (W - \mu) = \mu + \rho (W - \mu), \text{ when } \sigma_E = \sigma_W. \quad (5)$$

Similarly we also have the expectation of $W$ given the level of $E$.

The two countries have the same endowment distribution along each talent dimension, the only difference is the correlation. For any two agent $i, j$, with talent levels $E_i$ for $i$ and $W_j$ for $j$, they show up with the same probability in two countries. In the high-correlation country A, the corresponding talent levels along the other dimension for these two agents $i, j$ are expected to be: $\xi(W_i|E_i) = \rho_A (E_i - \mu) + \mu$ and $\xi(E_j|W_j) = \rho_A (W_j - \mu) + \mu$; In the low-correlation country B, the corresponding talent levels for these two agents are expected to be: $\xi(W_i|E_i) = \rho_B (E_i - \mu) + \mu$ and $\xi(E_j|W_j) = \rho_B (W_j - \mu) + \mu$, where $\rho_A > \rho_B$.

Given $E_i$ and $W_j$, the (expected) talent ratio for the other potential talent bundle for this team is

$$R \equiv \frac{E_j}{W_i} = \frac{\rho (W_j - \mu) + \mu}{\rho (E_i - \mu) + \mu}. \quad (6)$$

We are interested in the role of $\rho$ in this ratio, taking derivative with respect of $\rho$, we have

$$R' = \frac{\partial E_j}{\partial \rho} = \frac{(W_j - E_i)\mu}{[\rho (E_i - \mu) + \mu]^2}. \quad (7)$$

When $E_i > W_j$, we have $\frac{E_i}{W_j} > R$ for any value of $\rho \leq 1$, and $R' < 0$, thus the high-correlation country A has another bundle of smaller $E/W$ ratio than country B. As a result, the range of talent ratios is larger in country A than in country B.

When $E_i < W_j$, we have $\frac{E_i}{W_j} < R$ for any value of $\rho$; and $R' > 0$, thus the high-correlation country A has another bundle of higher $E/W$ ratio than country B. As a result, the range of talent ratios is again larger in country A than in country B. The rest of the proof mimics the Theorem 3 in OT.

At the end, with identical talent distribution along each single dimension, the high-correlation country has comparative advantage in industries with extreme talent intensities while the low-correlation country has comparative advantage in middle $E/W$ intensities. $\Box$

Thus even with different definition of correlations, both my model and OT find an important role for the higher moment of endowment distributions in international trade. This point will be further emphasized in the next part about the effect of diversity on trade.

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32To see this point, $\frac{E_i}{W_j} - R = \frac{\rho (E_i^2 - W_j^2 + (\mu - \rho \mu)(E_i - W_j))}{W_j (\rho (E_i - \mu) + \mu)}$, thus $\frac{E_i}{W_j} - R > (\leq) 0 \iff E_i > (\leq) W_j$. 

37
3.2.6 Diversity and Trade

The role of endowment diversity has been shown by GM by assuming two different production technologies: the super-modular and sub-modular technology. This part investigate the role of endowment diversity in this new framework.

**Theorem 3.** In an open economy of two countries, suppose they have the same aggregate endowment along every talent dimension. The only difference is the diversity (variance) of the endowment distribution along certain dimension. Then the high-diversity country has comparative advantage in industries with extreme (high or low) intensities of that dimension, while the other has comparative advantage in industries with middle intensities of that dimension.

**Proof.** Consider the case of two countries with the same aggregate endowment along all dimensions, same diversity along the $W$ dimension, and different diversity along the $E$ dimension. Compare the change of equilibrium from the low-diversity country (A) to the high-diversity country (B).

Under the random matching mechanism, agent matching outcome will be the same along the $W$ dimension. For any team in country A with agents endowment $(E_i, W_i)$ and $(E_j, W_j)$, there is a team in country B with agents endowment $(E'_i, W_i)$ and $(E'_j, W_j)$, where $E_i + E_j = E'_i + E'_j$, and $(E'_i - E'_j)^2 > (E_i - E_j)^2$. Then the effective talent bundles in country A are $(E_i, W_j)$ or $(E_j, W_i)$, and $(E'_i, W_j)$ or $(E'_j, W_i)$ for country B. Without loss of generality, assume $E_i > E_j$ and $W_i > W_j$, then $E'_i > E_i, E'_j < E_j$ the effective talent ratios has the following relationship:

$$\frac{E'_i}{W_i} < \frac{E'_j}{W_j} < \frac{E_i}{W_i} < \frac{E_j}{W_j};$$

Thus in country B, the effective bundles have more extreme ratios than those in country A. As a result, the high-diversity country B has comparative advantage in those industries with extreme talent intensities.

Under the optimal matching mechanism, again the proof is dealing with the PPFs in these two countries. For any pair of agents in country A with agents endowment $(E_i, W_i)$ and $(E_j, W_j)$, if there is a dominating choice of effective bundle, when $E_i > E_j$ and $W_i < W_j$ ($E_i < E_j$ and $W_i > W_j$), then effective bundle will be $(E_i, W_j)$ ($(E_j, W_i)$) in country A and $(E'_i, W_j)$ ($(E'_j, W_i)$) in country B. Country B has comparative advantage in industries with high (low) $E/W$ intensities. When $E_i > E_j$ and $W_i > W_j$, there is no dominating choice of effective bundle. Consider the output possibility of two goods, one with a high $E/W$ intensity $\alpha$ and

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\[33\] If instead $E_i > E_j$ and $W_i < W_j$, then $(E_i, W_j)$ is the dominant choice for any industry. In that case, we still have $\frac{E_i}{W_j} < \frac{E'_i}{W_j}$. 

38
the other with modest $E/W$ intensity $\beta$. Country A chooses $(E_i, W_j)$ to produce the high $\alpha$ good, chooses $(E_i, W_j)$ when $E_i^\beta W_j^{1-\beta} > E_j^\beta W_i^{1-\beta}$ or $(E_j, W_i)$ when $E_i^\beta W_j^{1-\beta} < E_j^\beta W_i^{1-\beta}$ to produce the $\beta$ good. For country B, $(E'_i, W_j)$ is always chosen to produce the $\alpha$ good, it chooses $(E'_i, W_j)$ when $E_i'^\beta W_j^{1-\beta} > E_j'^\beta W_i^{1-\beta}$ or $(E'_j, W_i)$ when $E_i'^\beta W_j^{1-\beta} < E_j'^\beta W_i^{1-\beta}$. Notice that when country A chooses $(E_i, W_j)$ to produce the $\beta$ good, B will choose $(E'_i, W_j)$, in this case, country B has comparative advantage in the high $E$ intensity $\alpha$ good; When country B chooses $(E'_i, W_j)$ and country A chooses $(E_j, W_i)$ to produce the $\beta$ good, this comparative advantage is amplified; When country B chooses $(E'_j, W_i)$ to produce the $\beta$ good, country A will choose $(E_j, W_i)$, country A has comparative advantage in this $\beta$ good. Analogously, in case of two goods with one modest intensity and one high $W/E$ intensity, similar results will be obtained, the high diversity country will have comparative advantage in the industry with high $W/E$ intensity and the low-diversity country has comparative advantage in the industry with modest intensity.

This theorem can be easily extended in the following way.

**Proposition 3.** In an open economy of two countries, assume same aggregate endowment along every talent dimension. One country has more diverse endowment distributions along one or more dimensions than the other does. Then the high-diversity country has comparative advantage in industries with extreme (high or low) intensities of that or those dimensions, while the other has comparative advantage in industries with middle intensities of that or those dimensions.

**Proof.** This proposition directly follows from Theorem 3. When there is a second dimension with higher diversity, it generates comparative advantage in industries with extreme intensities in this second dimension. This goes the same way with the first high-diversity dimension, since the extremely high relative intensity of one dimension corresponds to the extremely low relative intensity of the other dimension.

Compared to GM, similar results on the role of endowment diversity are generated here. In GM, the high-diversity economy has comparative advantage in industries featuring sub-modular production technology where the complementarity between tasks is lower, while the low-diversity economy has comparative advantage in industries featuring super-modular technology where the complementarity between tasks is higher. On the other hand, in my framework, industries differ in their task/talent intensity instead of task complementarity. The high-diversity economy has comparative advantage in producing industries with extreme talent intensities, while the low-diversity economy has comparative advantage in producing goods with middle talent intensities. The U.S. has a more diverse skill (talent) distribution.
compared with Japan. Thus U.S. has comparative advantage in those industries requiring very intensive usage of certain talent, acting like super stars on a team. While Japan’s comparative advantage lies in those industries require similar levels of various skills.

Again when compared with the model in OT, the endowment inequality comes from the increase of variances of $s$ and $l$. Increase of $s$ variance means more bundles with extreme talent ratios, while the increase of $l$ variance with correlation held constant results in more relative production of high-$s$ goods. The diversity here denotes variances of different dimensions of talent endowment. It works similarly with the variance of $s$ in OT.

3.3 The New Gains From Trade

There are new gains from trade attributed to the changes of the effective endowments in each country through two channels. First, when there is no dominant agent matching scheme, then optimal agent matching contains a proper mix of different agent matching schemes. The effective endowment can be changed by using a different mix of schemes. Moreover, assuming decreasing cost to scale for agent re-matching, the production possibility set is non-convex when there is no dominating agent matching scheme as discussed in the baseline model. This leads to another type of gains similar with the gains from trade in scale economy models.

Second, the effective endowment can be changed if for some teams there is no dominant choice of effective bundle for all industries. Then these teams may switch their effective bundles and change industries as good prices change when opening to trade.

In the general model with continuous endowment levels and a continuum of industries, there are several conditions required for these new gains to exist. I formally state them below.

**Assumption 1.** Agents have multidimensional endowment of talents; The production is performed by teams of completely specialized workers; There is a continuum of industries with different task intensities.

**Assumption 2.** There is no dominating matching scheme, thus the PPFs under different matching schemes have intersections.

**Assumption 3.** There is no dominating choice of effective talent bundle across industries for all teams along the PPF.

**Proposition 4.** Given assumptions 1, with either or both assumptions 2 and 3 hold, the new source of gains from trade will exist.
Proof. This proposition directly follows the intuitions shown in the baseline model. When assumptions 1, 2, 3 hold, there will always be potential to change the effective endowment of the economy to achieve a better utilization and allocation of the initial multi-dimensional endowments.

Proposition 5. Given proposition 1, the countries with higher endowment correlations have higher probability to obtain these new gains from trade.

Proof. With higher endowment correlation, the trade-off when choosing effective endowment bundles is relatively tied. Thus when outside conditions change, such as good price changes, there are more teams who want to change their effective bundles. Hence it is easier for this economy to gain from trade by changing the effective endowment.

The intuition of this proposition can be connected with mobility. High correlation endowment results in close trade-off between effective bundles, which differ in talent ratios. It is also saying these teams are more flexible in choosing effective bundles and industries. When outside conditions change, they are more mobile, thus also easier to gain from integration.

Proposition 6. As the production specialization increases, the number of asymmetric tasks, i.e. skill dimensions, increases. Given assumption 1, ceteris paribus, there is increasing probability for the extra source of gains to exist.

Proof. Given the assumption of multi-dimensional endowment, as the specialization of production increases, the dimension of talents increases, there are more potential choices of effective bundles. Hence there will be more potential to improve when outside conditions change upon trade integration.

3.4 Wage Inequality and Job Polarization

As shown in the baseline model, this new framework with multidimensional endowment and team production seems promising in explaining both the job polarization in all North economies and the different wage evolutions across the developed world.

Ever since the Roy model, multidimensional endowment has drawn attention from economists. Particularly, the pursuit of comparative advantage is shown to reduce the level of inequality from what would occur in a random assignment of workers into occupations. And this comparative advantage effect depends on the correlation between endowment dimensions. Gould (2002) finds evidence that the increasing correlation in US indeed contributed to the rising wage inequality.

\footnote{Each dimension could be a particular subset of different talents. The increase of dimensions gives increasing number of ways to utilize the multidimensional talents.}
In this new framework, endowment correlation has richer implications not only on the trade patterns as stated in the Theorem 2 but also on wage inequality and employment changes.

Multidimensional endowment has two effects on wage inequality. The selection effect decreases wage inequality when agents can choose their tasks according to their comparative advantages. The endowment also provide the option value when teams bargain on the shares of final output.

In the competitive case, assume that there are many potential firms that can freely enter the market and employ each team. Thus the two agents on each team get the total surplus of that team. In a competitive environment, there are also many teams with the same effective bundle in each industry, thus each agent on the team gets the share that he contributes to the total surplus. Particularly, in the Cobb-Douglas production case, each gets a share that equals his talent intensity.

**Proposition 7.** Given assumption 1, in the competitive environment, the economies with comparative advantage in extreme-intensity industries also experience increase in wage inequality; In contrast, the wage inequality decreases in economies whose comparative advantage in middle-intensity industries.

**Proof.** In the competitive environment, agents’ share on team surplus depends on their talent intensities. With more teams enter the extreme-intensity industries, the surplus division between team members becomes more uneven. Thus the wage inequality increases. The contrary applies to the economies with comparative advantage in middle-intensity industries. 

Blum (2008) finds that changes in the sectoral composition of the economy were the most important force behind the widening of the wage gap, accounting for about 60% of the relative increase in wages of skilled workers between 1970 and 1996. This fact seems to be in line with the above proposition. Service sector is usually recognized as a sector where personal success is more important in final output. Assuming team production, in service sector there are more teams with one superman and one sidekick. The crucial skill is more intensively used than the other. And wage gap between the superman and the sidekick is large. Thus an expansion of the service sector may results in an increase in wage inequality.

**Proposition 8.** In an open economy with multidimensional endowment, assume that the industry intensities are biased toward certain dimension. The developed North has higher endowment along that dimension relative to the South. Then a “job polarization” employment pattern will exist in the North after integration.
Proof. When the industry intensities are truncated, i.e. biased toward certain dimension. For instance, in all industries the entrepreneurial talent is equally or more intensively used than the worker skill. And the South has relatively poor endowment in the entrepreneurial talent, thus relatively rich endowment in the worker skill. In the open economy, the North has comparative advantage in those high $E$-intensity industries. In the competitive environment, these industries contain agents who earn the highest and lowest wages in the North. After trade integration, these industries expand the most. Hence employment in these industries increases relative to that in industries with middle intensities.

In most of the developed economies, a relatively higher increase in the employment of low-wage and high-wage jobs, namely the “job polarization”, has been found over the last three decades (Goos et al., 2007, 2009). Most of the researches on this pattern has been focused on the closed economy (Autor et al., 2006, 2008). Only a few tries to consider the role of trade integration (Costinot and Vogel, 2010 and Monte, 2011).\footnote{Costinot and Vogel (2010) needs to assume a extreme biased technology change to generate job polarization in all the North economies. Monte (2011) shows the skill-biased technology and trade integration between identical countries can produce the same pattern.} The task approach used by Autor et al. is the most recognized framework to investigate the job polarization pattern. They assume that workers on those middle wage jobs mainly perform some routine tasks, which are substitutable by machines and computers. In fact, when Dustmann, Ludstech and Schönberg (2009) dissect the German wage structure, they find it may not be true that those middle-wage workers are the ones who mainly perform routine tasks.

Even though this employment evolution pattern, job polarization, has been found in most North economies. There is however not a common pattern of wage and inequality evolution. The stylized fact in the literature says that US has a higher inequality level than the EU. Also Autor et al (2006) also finds a “wage polarization” in the US labor market besides the job polarization. The wage increase for low-wage workers and high-wage workers is higher than that for middle-wage workers. They propose the task approach and predict wage polarization should accompany job polarization. However, there is no strong evidence showing that wage polarization also exist in other developed North economies. The upper-tail inequality does not increase in France or Japan. In Germany, below the median, the correlation between employment and wage changes is negative rather than positive in the US.

This new framework with multidimensional endowment seems to be able to reconcile these differences. Based on Proposition 8 I argue that the common pattern of job polarization in the North may be a result of the North-South trade integration. To explain the different wage evolutions across North countries, this model embeds the difference in endowment correlations. European countries and also Japan have very specialized workers due to
their education system and also labor market institutions. This low endowment correlation generates comparative advantage in middle-intensity industries, thus lower wage inequality and small or no wage polarization.

3.5 Discussion

In this section, I very roughly discuss several interesting implications of this model.

3.5.1 Educational Policy

As shown in the above analysis, multidimensional endowment plays an important role in trade and labor market. The endowment correlation and dispersion both matters. In each country, agents’ talent endowments are largely shaped by its education system, thus different educational policies may have profound influence on the economy.

For example, the Germany has a very skill-specialized vocational educational system as compared with a more general-skill education system in the US. Youths in Germany are divided into schools with very different missions at a very early age (around 10), some of them are then going to secondary schools to prepare for college, while others are going to schools prepared for various vocational trainings. This early division increases the specialization of the German labor force, and decreases the correlation of their skill endowments. In the Unites States however, general purpose technology and general training are more emphasized in most liberal arts schools. Thus the correlation of talent endowment across dimensions is very high in the US. Workers with more years of education outperform those with less education in almost every dimension. As shown in my model, this difference in correlations generates comparative advantage in the service industries with extreme skill/task intensities for the US, and in manufacturing with middle skill intensities for the Germany.

Given the set of available industries, educational policies should be set in particular ways to accompany one’s comparative advantage. Particularly, for countries with comparative advantage in middle-intensity industries, such as manufacturing, more diversified skill endowments are preferred. Hence education policies should encourage specialization. Government subsidies for vocational training, unemployment insurance and directed agent match are optimal choices. The reverse is true for other economies with comparative advantage in extreme-intensity industries.

As specialization increases, the dimension of talent/skills increases, an efficient education system should allocate more resources to diversify the talent endowments, in order to limit the waste of talents and increase the equilibrium effective endowments that are actually utilized in production.
3.5.2 Task Off-shoring

The baseline model discussed in section 2 can be easily extended to allow international teams, or off-shoring certain tasks. It is interesting to consider the changes of occupational structure in each economy and its effects on the wage structure.

Consider the integration of a type-I North economy and a type-I South economy, if under autarky the self-matching scheme dominates. Then the effective bundles are: half \((L, L)\) and half \((H, H)\) in the North economy, half \((L', L)\) and half \((H', H)\) in the South economy. When open to trade, the North specialize in the \(E\) intensity good and South specialize in the \(W\) intensive good.

When agents in these two economies are allowed to form international teams, the optimal matching includes task off-shoring. North teams with effective bundle \((L, L)\) and South teams with effective bundle \((L', L)\) will break up. Each North agent with endowment \((E, W) = (L, L)\) is matched with a South agent with endowment \((E, W) = (L', L)\), each new team has effective bundle \((L, L)\), all North agents are doing the entrepreneurial task and South agents are doing the worker task. In the world economy, the effective bundles are half \((H, H)\), half \((H', H)\) and a unit of \((L, L)\). It is a welfare improvement allowing task off-shoring. In the end, there are more entrepreneurs in the North and more workers in the South.

Eeckhout and Jovanovic (2012) builds a model to investigate the occupational choice and development. Their predictions on occupational choice are similar with the model in this paper. And they do find support from the cross-country data that after integration, the high-skill countries see a disproportionate increase in managerial occupations and low-skill countries see an increase in wage work occupations.

3.5.3 Trade and Inequality

The effect of trade integration on income inequality, particularly in developing countries, still puzzles economists. Standard theories predict that inequality increases in the North developed economies, but decrease in the South developing economies. However, empirical evidences for this effect of trade on income distribution are mixed.

This paper emphasizes the role of industrial structure in shaping the income distribution. As economies develop, they experience different stages. Industrial specialization moves from manufacturing to service. As Blum (2008) has pointed out the rise of service sector is the most important contributor to the increase of US wage inequality between 1970 and 1996. When open to trade, the structural change is accelerated in the North. Thus inequality increases. The South also specialize in some industries with extreme high intensities in its

\[36\] This case exists when \(H/L\) is not too big and talent intensities are not too extreme.
abundant talent, the inequality level may not decrease as the standard trade model predicts.

4 Conclusions

This paper constructs an open economy model with multidimensional endowment and team production to explain the trade patterns and wage evolutions in the US and Europe during the globalization era. The effects of endowment correlation between dimensions and skill dispersion along each dimension on the trade patterns and labor market outcomes are investigated, in the case of North-North trade and North-South trade. The implications of this model are broadly in line with the empirical facts. The main findings are as follows.

First, the higher moments of the skill distribution, the endowment correlation and the dispersion in particular, play a important role in shaping the comparative advantage besides the relative aggregate endowments. With the same aggregate endowments in each dimension, the high-correlation (high-dispersion) country has comparative advantages in industries with extreme-value task intensities; the low-correlation (low-dispersion) country has comparative advantages in those middle-intensity industries. Moreover, the endowment distribution in each country is shaped by its educational system. Given similar aggregate endowment, the general-skill education system in the US and the skill-specific vocational education system in continental Europe result in a higher endowment correlation for agents in the US, which further generates comparative advantages in industries with extreme task-intensities. A higher dispersion in the US amplifies this effect. This simple model is able to explain the different trade patterns and wage inequality between the US and Europe, highlighting the effects of different educational policies in these two regions.

Second, it is the effective endowment instead of the initial endowment that determines the equilibrium in each economy. The effective endowment can be adjusted by reshuffling the agent-matching outcome or switching the task assignments within teams. These potential adjustments of the effective endowment provide a new source of gains from trade that differ from those conventional gains from trade identified in the literature.

Third, this model is able to generate a universal job polarization pattern in all developed countries along with different wage evolutions across the US and Europe as empirically found. Particularly, this model generates job polarization in the North economies in the case of North-South trade under reasonable assumptions. This is in line with the fact that the job polarization pattern took place roughly in the era of globalization, when the South economies open up to trade with the North. Hence this model indicates a role for globalization behind the job polarization pattern, complementing the existing explanation based on routine-task substitute technology changes in the labor literature. Furthermore, this model
also emphasizes divergent wage evolutions across North economies accompanying the common job polarization pattern. It suggests that the absence of wage polarization in Europe, but not in the US, may be a result of the lower endowment correlation and lower skill dispersion due to its vocational education system compared to the general-skill education in the US.

This framework can be adopted to investigate many interesting questions. Several applications of the model have been considered in this paper, such as the educational policy, task off-shoring and the effect of trade on wage inequality. There are also interesting empirical implications that might lead to promising empirical exercises when proper datasets are available. For example, in the case of North-South trade, the best managers and worst workers in the North benefit the most. Thus within the broad manager occupation, inequality increases. On the other hand, the inequality within the worker occupation decreases. The reverse is true for the South economy. These qualitative implications calls for more rigorous empirical investigations in the future.

The current model is also simplified in several ways and can be further extended. The agent matching process in this model is not specified in detail given the purpose of current paper is to determine the trade patterns. In reality agents are able to direct their searches towards particular jobs based on their endowments and the matching outcome may differ. The search model is worth investigation because it links to possible labor and trade policy interventions to improve efficiency. Unlike the adjustment costs discussions of the earlier literature (Leamer (1980), Feenstra and Lewis (1994), etc.), inefficiencies related to team production and matching appear to be long lasting and thus especially suitable targets for policy. Moreover, the team production organization is also extremely simplified. The team size is fixed. In contrast, many papers have pointed out that the size of the production unit is usually bigger in manufacturing than in services (Buera and Kaboski (2012) for example). The size of teams may also be endogenously chosen by different firms. A more detailed and micro-founded modelling of the team production seems promising and will be pursued in my future research.

References


