Vertical FDI and exchange rates over the business cycle: the welfare implications of openness to FDI

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Abstract

It has long been observed that a country tends to receive increased foreign direct investment (FDI) inflow when its currency depreciates. We provide an explanation of this correlation and examine the implied welfare effects of short-run FDI flow. Firms’ cross-border production location decisions are analyzed in an open-economy macroeconomic model in which both the exchange rate and FDI flow are endogenously determined. The exchange rate and FDI flow are positively correlated under monetary or productivity shocks. Furthermore, we show that short-run FDI fluctuations exacerbate utility loss over business cycles in an environment with nominal shocks, but has little impact on welfare over business cycles caused by productivity shocks. The first best outcome occurs when the economy retains long-run FDI, but restricts short-run movements in the production location of firms.

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“Ideally, you’d have every plant you own on a barge to move with currencies and changes in the economy.”
- Jack Welch

1 Introduction

From a macroeconomic perspective, foreign direct investment (FDI) is usually seen as a win-win strategy. It promotes economic growth for developing countries, and reduces production cost for developed countries. Economists have gone a long way in identifying and quantifying the various benefits of FDI. Previous studies mainly focus on the long-run equilibrium gain of openness to FDI. This paper adds to the literature by examining the welfare implications of openness to FDI from a short-run perspective. Our model features two countries that are asymmetric in average productivities. FDI flow that occurs in the steady-state of the economy is termed “long-run FDI,” and changes in the FDI flow over the business cycle is termed “short-run FDI.” We find that short-run FDI hurts world welfare under nominal shocks, but has little effect on welfare under productivity shocks. Overall, fluctuations of FDI over the business cycle reduce welfare, and the first best outcome is achieved when long-term FDI is retained while short-run movements in the production location of firms are prohibited. In addition, our model is able to account for an important stylized fact about FDI flows over the business cycle. It has long been observed that the net FDI inflow and exchange rate are positively correlated over the business cycle. Namely, a country tends to receive more FDI inflow when its currency depreciates. This pattern has been neglected by most previous general equilibrium models with multinational production. Our model produces this positive correlation between FDI inflow and exchange rate under both monetary and productivity shocks.

In recent years, the popular press has noted the return of manufacturing jobs to developed countries following the depreciation of domestic currencies.\footnote{See, for example, “Canon to return some production to Japan,” The Wall Street Journal, Jan. 29, 2014, and “Welcome home,” The Economist, Jan. 19, 2013.} This relationship between exchange rate and FDI flow has been well known to economics researchers. Since the mid-1980s, a large volume of empirical studies has documented that a country’s exchange rate and FDI inflows are positively correlated. The abundance of empirical evidence has led Blonigen (2005), in a survey paper about the empirical determinants of FDI, to list the
level of exchange rate as one of the significant determinants of short-run FDI flow. Earlier theories explaining this correlation typically take a partial equilibrium approach in which the movement in exchange rate is treated as an independent and exogenous event. This paper provides an explanation to this correlation by incorporating multinational enterprises (MNEs) into an open economy macroeconomic model where both the level of FDI flow and exchange rate are endogenously determined. We then examine the welfare effects of FDI flow in this framework.

We study firms’ production location decisions in a two-country model based on Devereux and Engel (2001). Our paper focuses on vertical FDI flows, which means that firms move to produce in a foreign country to take advantage of lower production cost abroad, rather than to gain access to consumers of the local market. FDI is modeled as firms’ prepaid, fixed investment cost when they decide to move production overseas. Location decisions need to be made one period ahead of actual production. Firms in each country are heterogeneous in their productivity levels, and the two countries are asymmetric in terms of average productivity. Households set wage in a Calvo manner, inducing sluggish adjustment of the real wage. The uncertainty in the model comes from both monetary shocks and productivity shocks.

A firm’s relative profitability of producing abroad depends positively on its productivity, and on the expected gap in real wages between domestic and foreign workers. A firm decides its production location by comparing the expected gain from switching to a low production cost location with the fixed cost of the switch. Given a lower expected real wage abroad, there exists a cutoff firm who will just break even in expectation by switching its production location. Every firm with a higher productivity than the cutoff firm must earn a strictly positive profit abroad, and thus becomes a MNE in the following period.

We show that the countries’ average productivities determine the long-run direction and volume of cross-border FDI flow. Specifically, the country with higher average productivity becomes the source country of FDI, and the country with lower productivity becomes the recipient country. This is because without cross-border relocation of production, the country with lower productivity has lower real wage, making it profitable for the more productive firms in the source country to relocate.

The model generates positive comovement between exchange rate and FDI under either monetary or productivity shocks. A domestic monetary expansion, for example, causes a depreciation of domestic real wage under sluggish wage adjustment. As the wage gap is expected to persist into the next period, when production takes place, more firms are enticed

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2This empirical fact is robust for developed countries and developing countries alike. A non-exhaustive list of empirical works confirming the positive relationship include Cushman (1985); Ray (1989); Froot and Stein (1991); Klein and Rosengren (1994); Grosse and Trevino (1996); Bayoumi and Lipworth (1998); Kiyota and Urata (2004); Buch and Kleinert (2008).
to relocate to the depreciating country. On the other hand, a positive domestic productivity shock causes domestic real wage to increase relative to foreign wage. The lower expected relative real wage abroad, together with better production technology, increases a firm’s expected gain from moving abroad. Thus more firms are prompted to produce in the foreign country.

The steady state of the model is Pareto optimal. Consequently, deviations from steady-state following a nominal shock represent inefficiencies caused by nominal rigidity. In this process, firm relocation tends to further aggravate the harm. Consider a foreign depreciation. The influx of foreign firms increases labor demand, but without FDI, foreign households are already working too much compared with the flexible-wage long-run equilibrium level. Yet when cross-border relocation is allowed, the increased labor demand from the new MNEs causes foreign households to work even more. Likewise, domestic households will work even less. Thus when the economy is hit by monetary shocks, firm relocation pushes aggregate employments further away from efficient levels.

To assess how FDI affects welfare, we compute the expected world utility by numerically simulating the model under second order approximation. We compare expected world utility in the benchmark model with utilities in two alternative scenarios - when all firms produce domestically, and when only long-term relocations are allowed. Our key findings are three-fold. First, firm relocations in the steady-state are welfare-improving. Second, with nominal shocks, short-run FDI fluctuations further deteriorate welfare loss caused by nominal frictions through the aforementioned employment effects. With real shocks, short-run FDI fluctuations have little welfare effect as a result of several counteracting forces. Finally, the first best outcome can be achieved in a setting where long term firm relocation is retained, but short-run, temporary relocations are eliminated. In the benchmark simulation, world utility is highest at every volatility level of real and nominal shocks when the distribution of MNEs is kept constant at the steady-state level, but moving further back-and-forth is prohibited in the short run.

This paper contributes to the literature on gains from openness to FDI. Previous works primarily focus on long-run gains from productivity improvement. For instance, McGrattan and Prescott (2009) examine host country’s gains from FDI in a growth model in which a country can exploit another country’s technological capital by opening to FDI. In the Burstein and Monge-Naranjo (2009) model, foreign management know-how is incorporated as an additional factor of production into a neoclassical model. The authors estimate that the host country has large output and welfare gains from opening to FDI. Ramondo and Rodríguez-Clare (2013) and Ramondo (2014) employ models based on new trade theory to measure gains from FDI. Ramondo (2014) focuses on horizontal FDI, in which affiliates of
MNEs sell output only in the host country. Ramondo and Rodríguez-Clare (2013), on the other hand, incorporates various types of FDI into a trade model, and uses the model to examine how trade interacts with FDI. These papers all look at long-run gains from openness to FDI derived from increased productivity or availability of goods. Our work contributes to the literature by providing a welfare analysis from a short-run perspective.

Early theories explaining the positive relationship between FDI inflow and exchange rate include, but are not limited to, Froot and Stein (1991) and Blonigen (1997). Most early theories take a partial equilibrium approach in which exchange rate is assumed to be exogenous. A notable exception is Russ (2007), in which both the exchange rate and FDI are endogenously determined. Russ (2007) is a study of horizontal FDI, where firms relocate to serve consumers in a foreign market, and the focus is on the relationship between the volatility of exchange rate and FDI. Our model however focuses on vertical FDI. We don’t impose the market segregation that is necessary to motivate horizontal FDI. Instead, our framework features fluctuations in the relative real wage that induce changes in firms’ optimal production locations, which is missing from her model.

Our study is also related to a recent and booming literature that analyzes the dynamics of international macroeconomic models with multinational production. Burstein et al. (2008) and Arkolakis and Ramanarayan (2009) look at how vertical integration of countries affect their business cycle synchronization. Motivated by observations of US multinational activity in Mexico, Bergin et al. (2009, 2011) argue that vertical FDI increases employment volatility of the recipient country, relative to the source country. Cavallari (2010, 2013) incorporate firm entry into an international macro model, and show that the models’ predictions better match international business cycle moments in terms of consumption and investment spillovers as well as output comovement.

The rest of the paper is organized as follows. We set up the households’ and firms’ problems and describe the environment of the model in Section 2. We examine the long-run equilibrium of the economy by looking at the non-stochastic steady-state in Section 3. Section 4 studies the first order dynamics of the model. We first solve the model analytically under a specific assumption to gain insights into the model dynamics, and then we solve the model using numerical simulation to further dissect the details in the general case. Section 5 analyzes welfare effects of firm relocation. Section 6 concludes.

2 Model

There are two countries, North and South, each inhabited by a continuum of households whose total mass is normalized to 1. That is, the world population is 2. In addition, there
is a continuum of firms whose total mass is also 2. We assume that half of these firms are owned by Northern households and the other half by Southern households. The only inherent difference between North and South is the average productivity of firms. Each firm is a monopolistic supplier of a differentiated good, and the full nature of these firms will be specified later.

2.1 Households’ problem

A Northern household $h$ tries to maximize a time-separable utility function

$$E_t \sum_{j=0}^{\infty} \beta^j u(C_{t+j}(h), N_{t+j}(h)).$$

The period utility function depends on consumption of a composite good $C_t(h)$ and labor supply $N_t(h)$:

$$u(C_t(h), N_t(h)) = \frac{1}{1-\sigma} C_t(h)^{1-\sigma} - \frac{1}{1+\phi} N_t(h)^{1+\phi} \equiv U(C_t(h)) - V(N_t(h)),$$

where the parameters satisfy $\sigma > 0$ and $\phi > 0$. The composite good is a Dixit-Stiglitz CES aggregate of differentiated individual consumption goods defined as

$$C_t(h) = \left[ \int_0^2 C_t(h, f)^{\frac{\mu-1}{\mu}} df \right]^{\frac{\mu}{\mu-1}},$$

where $C_t(h, f)$ is household $h$’s consumption of the differentiated good produced by firm $f$.

We assume that the elasticity of substitution $\mu$ is greater than 1. Each Northern household $h$ chooses its consumption basket taking the set of individual goods’ prices $P_t(f)$ as given.

We follow the approach of Erceg et al. (2000) to incorporate a Calvo type wage-setting mechanism. We assume that households are monopolistic suppliers of labor. Each household supplies a differentiated type of labor to the firms. Firms produce final products using the “composite labor” which is a CES aggregate of differentiated types of labor defined as

$$L_t = \left[ \int_0^1 N_t(h)^{\frac{1+\eta}{1+\eta}} dh \right]^{1+\eta}.$$
labor, is given by
\[ W_t = \left[ \int_0^1 W_t(h)^{-\frac{1}{\eta}} dh \right]^{-\eta}, \] (1)
where \( W_t(h) \) is the wage rate set by household \( h \) for its differentiated type of labor. Like any individual firm, each household \( h \) has zero weight in the continuum of households, and therefore its particular wage rate has no effect on the aggregate wage. Thus any individual household sets a wage taking the aggregate wage rate as given. Like the monopolistically competitive firm, each household faces a downward-sloping labor demand curve for its particular type of labor given by
\[ N_t(h) = \left( \frac{W_t(h)}{W_t} \right)^{-\frac{1+\eta}{\eta}} L_t, \] (2)
where \( L_t \) is the aggregate employment in period \( t \).

Households engage in Calvo-type wage setting. In each period \( t \), with probability \( (1 - \theta) \), household \( h \) is able to update the wage rate it offers. Otherwise, its wage rate will be \( \Pi W_{t-1}(h) \), where \( \Pi \) is the unconditional long-run gross rate of inflation of the economy, and \( W_{t-1}(h) \) is the wage rate household \( h \) charged last period.

There is an integrated world financial market where a complete set of state-contingent nominal bonds are traded. These bonds are (arbitrarily) denominated in the Northern currency. Thus, household \( h \) faces the recursive period budget constraint
\[ P_t C_t(h) + \sum_{\nabla^{t+1} \in \Omega^{t+1}} Z(\nabla^{t+1}|\nabla^t) D(h, \nabla^{t+1}) \leq (1 + \tau_w)W_t(h)N_t(h) + \Gamma_t + T_t + D(h, \nabla^t), \] (3)
where \( D(h, \nabla^{t+1}) \) is the units of nominal bond household \( h \) acquires. Each of these bonds pays one unit of Northern currency in period \( t + 1 \) in state \( \nabla^{t+1} \). The period \( t \) price of such a bond is denoted \( Z(\nabla^{t+1}|\nabla^t) \). The set of all possible states in period \( t + 1 \) is denoted \( \Omega^{t+1} \). \( P_t \) is the consumer price index. \( \Gamma_t \) is the profit earned by Northern firms that is distributed to the household lump-sum, \( T_t \) is government transfer, and \( \tau_w \) is the rate of government subsidy to labor.

As Ricardian equivalence holds, we assume that the government has a balanced budget every period without loss of generality. Government in this model subsidizes domestic workers and firms at rates \( \tau_w \) and \( \tau_p \), respectively, in order to offset monopolistic distortion. Thus the total government transfer (or negative of tax) to domestic households equals to negative
of the expenditure on labor and firm subsidies:

\[ T_t = -\tau_w \int_0^1 W_t(h)N_t(h)dh - \tau_p \int_0^1 P_t(f)Y_t(f)df, \]

where \( Y_t(f) \) is the output of firm \( f \). We assume that the monetary authority sets nominal interest rate \( i_t \) according to an inflation-targeting rule

\[ i_t = \bar{i} + \rho(\pi_t - \bar{\pi}) + \nu_t, \rho > 1, \nu_t \sim N(0, \sigma^2_\nu), \tag{4} \]

where \( \bar{i} \) is the steady-state nominal interest rate, \( \pi_t \) and \( \bar{\pi} \) denote the period \( t \) and steady-state inflation rates respectively, and \( \nu_t \) is the exogenous i.i.d. monetary disturbance.

Intra-temporal utility maximization gives us the familiar consumption price index \( P_t \), defined as the minimum cost of acquiring one unit of the composite consumption good:

\[ P_t = \left[ \int_0^2 P_t(f)^{1-\mu}df \right]^{\frac{1}{1-\mu}}, \]

while the demand for an individual product \( f \) is given by

\[ C_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\mu} C_t, \tag{5} \]

where \( C_t \) is the aggregate Northern consumption.

Household \( h \)'s first order conditions from the utility-maximization problem include

\[ Z(\nabla^{t+1}|\nabla^t) = \Pr(\nabla^{t+1}|\nabla^t) \beta \frac{[C_t(h)]^\sigma P_t}{[C_{t+1}(\nabla^{t+1}, h)]^\sigma P_{t+1}}, \tag{6} \]

which, when summed across all states, gives us the Euler equation

\[ Z_t = \beta E_t \left[ \frac{C_t^\sigma P_t}{C_{t+1}^\sigma P_{t+1}} \right]. \]

In the previous equation, \( Z_t \) is the price of the riskless nominal portfolio that pays one unit of Northern currency in every state at time \( t + 1 \), i.e. the inverse of the gross nominal interest rate. The existence of complete market allows us to drop the household index in consumption, assuming initial symmetry.

We assume that the government chooses the subsidy rate \( \tau_w = \eta \) to counteract the monopolistic distortion caused by the market power of wage-setters. The first order condition
regarding the optimal wage-setting is then

\[
E_t \sum_{j=0}^{\infty} (\beta \theta)^j \left[ V_{N(h),t+j} - \frac{W_t(h) \Pi^j}{P_{t+j}} U_{c,t+j} \right] N_{t+j}(h) = 0. \tag{7}
\]

where \( V_{N(h),t+j} = (N_{t+j}(h))^\phi \) is the marginal disutility of labor for household \( h \) in period \( t + j \), and \( U_{c,t+j} = C_{t+j}^{-\sigma} \) is the marginal utility of consumption. The conditional expectation is taken over states in which the household is unable to update its wage. When a household gets the chance to update its wage in period \( t \), it takes into account the fact that with probability \( \theta \) every period, the current reset wage will remain effective into the future. Thus it optimally sets a wage by weighing the discounted sum of future marginal disutility of working against the marginal utility of consumption made possible by the extra income from working. As the individual labor demand function (2) makes clear, household \( h \)'s reset wage at period \( t \) depends only on the current and expected future aggregate variables. Thus all wage-resetters in period \( t \) must set the same wage rate \( W_t(h) = \tilde{W}_t \). Given that a constant fraction \( (1 - \theta) \) of households reset wage in every period, the aggregate wage rate evolves according to

\[
W_t = \left[ (1 - \theta) \tilde{W}_t^{-\frac{1}{\eta}} + \theta (\Pi W_{t-1})^{-\frac{1}{\eta}} \right]^{-\eta}. \tag{8}
\]

Southern households are symmetric, and Southern variables are denoted by an asterisk over their Northern counterpart.

### 2.2 Firms’ problem

Each firm produces a differentiated product using a technology that is linear in the input of the composite labor. A Northern firm who produces domestically has the production function

\[
Y_t(\varphi) = \varphi A_t L_t(\varphi), \tag{9}
\]

where \( \varphi \) is the time-invariant, firm-specific productivity, and \( A_t \) represents the stochastic aggregate productivity of Northern firms. \( Y_t(\varphi) \) and \( L_t(\varphi) \) denote the output and labor hired by firm \( \varphi \), respectively. As firms in a given country only differ in terms of their productivity levels, we index Northern firms by their productivities \( \varphi \) from now on.

Assume each country gets an aggregate, country-specific productivity shock. The shock affects all firms owned by residents of this country in completely symmetric ways. An increase in \( A_t \) could capture such technology innovations as the use of internet and/or managerial improvements in a country.
Note that aggregate productivity affects firms headquartered in the country, rather than laborers who reside in the country. In models without firm relocation, these are equivalent. But when firms are allowed to move across border, it becomes an important distinction. We think in our context, firm-augmenting technology is more realistic and reasonable because, when considering cross-border FDI flow, firms are typically the patent holders. This is also consistent with the literature on welfare gains from multinational production, which emphasizes how MNEs take production technology and managerial know-how to their foreign subsidiaries.\(^3\)

We abstract from idiosyncratic firm productivity shocks. Our study focuses on macroeconomic outcomes of MNE production. Idiosyncratic shocks change firms’ relative positions in the productivity distribution, but as we will show below, the behavior of cross-border FDI flow is determined by the relative aggregate productivity of North and South.

Therefore, when a Northern firm produces as a multinational in the South, its production function is

\[
Y_{Nt}^*(\varphi) = \varphi A_t L_{Nt}^*(\varphi),
\]

where the subscript \(N\) is a reminder of the origin of the firm. Thus the firm takes its firm-specific technology and the home-country productivity with it to the South. The only component foreign in its production is the employment of Southern workers.

We assume the aggregate productivity follows the process

\[
\log A_t = \zeta \log A_{t-1} + \epsilon_t, \quad 0 < \zeta < 1, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2_\epsilon),
\]

where \(\epsilon_t\) is the exogenous i.i.d. productivity shock.

Firm heterogeneity in productivity is introduced to capture the empirical regularity that multinationals are found to be the more productive firms in their industry. They earn higher revenue, hire more workers, and make larger profit.\(^4\) Following Helpman et al. (2004) and Chaney (2008), productivities are assumed to be drawn from a Pareto distribution with shape parameter \(\alpha\). Then the Northern firm productivity cumulative distribution function is given by

\[
\Pr (\varphi' < \varphi) \equiv G(\varphi) = 1 - \left( \frac{\varphi_m}{\varphi} \right)^\alpha, \varphi \geq \varphi_m,
\]

where \(\varphi_m\) is the lower bound of the Northern productivity distribution. The shape parameter

\(^3\)Examples include, but are not limited to, Ramondo and Rodríguez-Clare (2013), McGrattan and Prescott (2009), and Burstein and Monge-Naranjo (2009).

\(^4\)For example, Helpman et al. (2004) examines the characteristics of multinational firms compared with exporters, as well as firms who only serve the domestic market. Their paper concludes that among these three sets of firms, multinationals have the highest average productivity, followed by exporters and then domestic firms.
\( \alpha \) is an inverse measure of the heterogeneity of firms. A lower value of \( \alpha \) indicates a fatter upper tail of the productivity distribution. We assume that \( \alpha > \mu - 1 \) to ensure that the \((\mu - 1)^{th}\) moment of the distribution is bounded.\(^5\) We then normalize the Northern minimum productivity \( \varphi_m \) to one, but assume that the South has a minimum productivity \( \varphi^*_m < 1 \). Thus Northern firms have, on average, higher productivity.\(^6\)

Firms’ production location decisions determine cross-border FDI flows. To produce in period \( t \), a firm has to first decide, in period \( t - 1 \), where to produce next period. The firm can choose to produce either in its domestic country or in the foreign country.\(^7\) If it decides to move its production facility overseas, it has to pre-commit a fixed real investment cost \( F \) in terms of the aggregate consumption good.\(^8\) Producing at home requires no additional cost. We assume that the location decision is irreversible by time \( t \), when production takes place. We also assume the location decision-making is repeated every period, and the fixed investment cost has to be paid each time when the firm decides to produce abroad in the following period.\(^9\) The volume of FDI flow in this model is defined as the total fixed investment taken by all multinational firms. That is, FDI inflow into the South in period \( t \) is \( m_tF \), where \( m_t \) is the mass of Northern firms who will produce as MNEs in the South in period \( t + 1 \).

Firms are optimally subsidized at the rate \( \tau_p = 1/(\mu - 1) \), such that for every unit of final product the firm sells, the firm receives \( (1 + \tau_p)P_t(\varphi) \). We assume that each country’s government subsidizes its own firms, even if the firm is producing abroad.

We abstract from goods market imperfections, and assume that goods are freely traded with no cost. Given this assumption, the law of one price holds for each individual good. The firm thus receives the same unit price on its sales in both countries, when expressed in

\(^5\)This is necessary because otherwise, the total revenue made by MNEs will be unbounded when the cutoff productivity goes to infinity.

\(^6\)An alternative assumption is to assume that the distribution of Northern firms has a different shape parameter \( \alpha \) than that of Southern firms. All qualitative results we present below only require that Northern firms are more productive on average and do not depend on how the difference in average productivity is introduced.

\(^7\)A firm can, in principle, produce in both countries. However, given the setup of the model, it does not make sense to have more than one production facility. It is always better to produce in the low cost location.

\(^8\)An alternative assumption in the literature is to assume that \( F \) is a fixed cost in units of labor. Adopting this alternative specification will only strengthen our results. As it will become clear, MNEs in the model relocate to the location with lower labor cost. If the fixed cost is in units of labor, then the fixed cost will decrease with the real wage, thereby enhancing firms’ incentive to relocate.

\(^9\)The assumption of a periodically repeated location decision is made mainly for tractability, but does not change the qualitative results of the model. The real wage gap that induces firms to relocate is persistent due to the sluggish adjustment of real wage and persistent productivity shocks. If we assume that the firm’s location decision is relevant for multiple periods, the decision rule will involve comparing a string of discounted future benefits with the fixed cost of relocation. While being more complicated, this alternative formulation does not change the qualitative nature of the decision.
a common currency. The firm’s total units sold worldwide is denoted $Y_t(\varphi)$.

Firms flexibly set prices of their outputs. Combined with the production function (9) and demand function (5), the firm’s profit maximization problem at time $t$ when producing domestically can be written

$$\max_{P_t(\varphi)} (1 + \tau_p) P_t(\varphi) Y_t(\varphi) - W_t \frac{Y_t(\varphi)}{\varphi A_t}$$

subject to the demand function

$$Y_t(\varphi) = \left( \frac{P_t(\varphi)}{P_t} \right)^{-\mu} Y_t^w,$$  \hspace{1cm} (10)

where $Y_t^w \equiv C_t + C_t^* + m_tF$ is the total world demand for the composite good. Note that as the fixed cost is assumed to be in units of the composite consumption good, period $t$ world demand is increased by the amount $m_tF$ when cross-border relocations are present.\(^{10}\) Since consumption utility functions are identical across countries, purchasing power parity (PPP) holds for the aggregate prices: $P_t = S_t P_t^*$. The nominal exchange rate $S_t$ is defined as the Northern currency price of the Southern currency. Therefore a firm’s relative prices in the two countries must be equal. This fact enables us to abbreviate the individual demand into (10).

As the result of the optimal subsidy, firms produce the optimal output, at which prices equal to their respective marginal costs. A Northern firm producing at home will optimally set price at

$$P_t(\varphi) = \frac{W_t}{\varphi A_t},$$  \hspace{1cm} (11)

and if it produces in the South instead, the optimal price reflects its Southern labor cost and Northern productivity

$$P^*_N(\varphi) = \frac{W^*_t}{\varphi A_t}. \hspace{1cm} (12)$$

Now we turn to a typical firm’s location choice problem. If a firm decides to produce in the foreign country, it has to pre-commit the fixed cost $F$. When a Northern firm with productivity $\varphi$ produces domestically, it earns the profit

$$\Gamma_{Nt}(\varphi) = \left( \frac{1}{\mu - 1} \right) W_t^{1-\mu} (\varphi A_t)^{\mu-1} P^*_t Y_t^w,$$  \hspace{1cm} (13)

\(^{10}\)We denote the total mass of MNEs producing in period $t$ by $m_{t-1}$. The $t - 1$ subscript reflects the one-period-ahead decision making on production location.
and if producing in the South, it earns the profit

$$\Gamma_{Nt}(\varphi) = \left(\frac{1}{\mu - 1}\right) (S_t W_t^*)^{1-\mu} (\varphi A_t)^{\mu-1} P_t^\mu Y_t^w, \quad (14)$$

expressed in Northern currency. The firm will choose to produce as a multinational if and only if the expected discounted gain from moving abroad is large enough to compensate for the current fixed cost $F$, i.e.

$$E_t [\delta_{t+1}(\Gamma_{Nt}(\varphi) - \Gamma_{Nt}(\varphi))] \geq P_t F,$$

where $\delta_{t+1} \equiv \beta P_t C_t^\sigma / P_{t+1} C_{t+1}^\sigma$ is the stochastic discount factor. When we substitute the profit functions (13) and (14) into the expression above, use the optimal prices (11) and (12), and rearrange, we can reduce the inequality to

$$\left(\frac{\beta}{\mu - 1}\right) \varphi^{\mu-1} E_t \left\{ \frac{C_t^\sigma}{C_{t+1}^\sigma} (A_{t+1})^{\mu-1} Y_{t+1}^w \left[ (\omega_{t+1}^*)^{1-\mu} - (\omega_{t+1})^{1-\mu} \right] \right\} \geq F, \quad (15)$$

where $\omega_t \equiv W_t / P_t$ denotes the Northern real wage and $\omega_t^* \equiv W_t^* / P_t^*$ denotes the Southern real wage. Inequality (15) outlines the break-even condition for Northern firm $\varphi$. The left-hand-side is the real expected discounted gain from switching production abroad, and the right-hand-side is the real cost.

The break-even condition (15) suggests three points. First, a lower real wage in the South, and therefore a lower marginal cost of production, is the source of potential gain for a Northern firm to become multinational. On the other hand, if Southern real wage is lower, no Southern firm will find it profitable to move to produce in the North. Thus there can only be one-way FDI flow in the model. Second, the scope of the gain is a monotonically increasing function of the firm’s productivity level, implying that there exists a cutoff productivity $\tilde{\varphi}_t$ such that in period $t$, a Northern firm with productivity $\varphi = \tilde{\varphi}_t$ will just break even in expectation by switching its production location to the South in period $t + 1$, and every Northern firm with productivity level $\varphi > \tilde{\varphi}_t$ must expect to make a positive net discounted profit by relocating and thus must decide to produce as a multinational in the next period. Finally, a higher expected aggregate productivity in the North makes Northern firms more likely to be a multinational, as it enhances the marginal gain from accessing lower cost labor in the same manner as the firm-specific productivity.

Southern firms have symmetric production functions and face the same choices. The only asymmetry between North and South is the average productivity levels of firms.
2.3 Aggregation

In this section, we aggregate across firms, so that each economy can be described as having a “representative firm” whose productivity depends on the MNE distribution. We write down the equations for the case when some Northern firms are producing in the South as multinationals. The case of Southern FDI flow into the North is symmetric to the current analysis.\footnote{We show later that given the assumption that Northern firms have higher average productivity, FDI flow from North to South is the relevant case to examine.}

Northern firms with productivity $\varphi < \tilde{\varphi}_{t-1}$ produce in the domestic country in period $t$. Using the production function (9), the demand for individual goods (5), and the optimal price-setting (11), we can write the aggregate labor demand as a function that depends only on aggregate variables and a measure of aggregate productivity in the North:

$$L_t = \int_{1}^{\tilde{\varphi}_{t-1}} \left( Y_t(\varphi) / \varphi A_t \right) g(\varphi) d\varphi = \omega_t^{-\mu} Y_t w A_t^{\mu-1} \varphi_{Nt-1}^{\mu-1},$$  

(16)

where

$$\varphi_{Nt-1} \equiv \left[ \int_{1}^{\tilde{\varphi}_{t-1}} \varphi^{\mu-1} g(\varphi) d\varphi \right]^{1/\mu-1}$$  

(17)

is a measure of the aggregate productivity of firms who produce in the North. Likewise, the labor demand in the South is given by

$$L^*_t = \int_{\tilde{\varphi}_{t-1}}^{\infty} \left( Y^*_t(\varphi) / \varphi^* A^*_t \right) g(\varphi) d\varphi + \int_{\varphi^*_m}^{\varphi^*_*} \left( Y^*_t(\varphi^*) / \varphi^* A^*_t \right) g(\varphi^*) d\varphi^*$$  

(18)

$$= (\omega^*_t)^{-\mu} Y^*_t w \left[ A^*_t^{\mu-1} \varphi^*_{Nt-1}^{\mu-1} + A^*_t^{\mu-1} \varphi^*_{Nt-1}^{\mu-1} \right],$$

where

$$\varphi^*_{Nt-1} \equiv \left[ \int_{\tilde{\varphi}_{t-1}}^{\infty} \varphi^{\mu-1} g(\varphi) d\varphi \right]^{1/\mu-1}$$  

(19)

is the aggregate productivity of Northern firms who produce in the South in period $t$, i.e., the multinationals, and

$$\varphi^* \equiv \left[ \int_{\varphi^*_m}^{\varphi^*_*} \varphi^{\mu-1} g(\varphi^*) d\varphi^* \right]^{1/\mu-1}$$

is the mean productivity of Southern firms.

Meanwhile, the total profit earned by Northern firms, denoted by $\Gamma_t$, includes profits of Northern firms who produce domestically, and Northern firms who produce in the South (in case multinational firms are present). These profits are reduced by the amount of the fixed
investment cost
\[ \Gamma_t = \Gamma_{Nt} + \Gamma^*_N - (1 - G(\bar{\varphi}_t)) P_t F. \]

Using the optimal price-setting rule (11), for a Northern firm \( \varphi \) who produces domestically, its profit is
\[ \Gamma_{Nt}(\varphi) = (1 + \tau_p) P_t(\varphi) Y_t(\varphi) - W_t L_t(\varphi) = \left( \frac{1}{\mu - 1} \right) W_t L_t(\varphi). \]

Integrating over domestically-producing Northern firms using a similar procedure as in (16), we find that the total profit made by these firms is given by, in real terms,
\[ \frac{\Gamma_{Nt}}{P_t} = \left( \frac{1}{\mu - 1} \right) (\omega_t)^{1-\mu} Y_t^{\mu-1} A_t^{\mu-1} \bar{\varphi}_{Nt-1}^{\mu-1}. \]  (20)

Likewise, the total real profit made by the set of Northern MNEs is
\[ \frac{\Gamma^*_N}{P_t} = \left( \frac{1}{\mu - 1} \right) (\omega^*_t)^{1-\mu} Y_t^{\mu-1} A_t^{\mu-1} \bar{\varphi}^{\mu-1}_{Nt-1}. \]  (21)

In the next section, we will use these aggregate profits to derive steady-state consumption.

### 3 Steady-State FDI

We will proceed by linearizing the model around a non-stochastic steady state, assuming zero initial wealth. The steady state describes the long-run equilibrium toward which the economy moves absent shocks. In the steady state, all real variables are constant and all nominal variables grow at the constant rate \( \bar{\pi} \). We denote the value of a variable at steady state by an overbar, except for the aggregate productivities (17) and (19), whose steady-state values are simply denoted by the variables without the time subscript.

As there can’t be two-way FDI flows, the direction of FDI flow in the steady-state could fall in one of three cases: Northern firms relocate to produce in the South, Southern firms relocate to produce in the North, or there could be no FDI flow. We write down the relevant equations for the first case here. The reversed case is symmetric, and the zero FDI case is a special case of the former two. We prove that given the assumption of higher productivity in the North, in the steady state, FDI flows from North to South.

In the non-stochastic steady-state, every Northern household sets the same wage rate and supply the same amount of labor, implying \( \bar{L} = \bar{N}(h) \) and \( \bar{W} = \bar{W}(h) \) across house-.
holds. The wage distribution degenerates into a single value. The optimal wage-setting rule (7) reduces to the simple and familiar labor supply rule that equates the marginal rate of substitution to the real wage
\[ \bar{\omega} = \bar{L}^{\phi} \bar{C}^{\sigma}, \] (22)
\[ \bar{\omega}^* = \bar{L}^*^{\phi} \bar{C}^*^{\sigma}. \] (23)

Dividing the Northern labor supply by its Southern counterpart, we get
\[ \frac{\bar{\omega}}{\bar{\omega}^*} = \left( \frac{\bar{L}}{\bar{L}^*} \right)^{\phi} \left( \frac{\bar{C}}{\bar{C}^*} \right)^{\sigma}. \] (24)

Incorporating steady-state versions of the aggregate labor demands (16) and (18) into (24), the labor market equilibrium in the steady-state thus requires
\[ \left( \frac{\bar{\omega}}{\bar{\omega}^*} \right)^{\frac{1}{\phi} + \mu} = \left( \frac{\bar{C}}{\bar{C}^*} \right)^{\frac{\phi}{\sigma}} \frac{\bar{\varphi}^{\mu-1}_N}{\bar{\varphi}_N^{\mu-1} + \bar{\varphi}^{*\mu-1}} , \] (25)
where $\bar{\varphi}_N$ and $\bar{\varphi}_N^*$ are the steady-state values of the aggregate productivities defined in (17) and (19).

The budget constraint states that in the steady-state, each country’s consumption is the sum of its residents’ labor income and the net profits earned by domestic firms, minus the government subsidy to firms. Thus for the North,
\[ \bar{C} = \bar{\omega} L + \bar{\Gamma}_t/P_t - \tau_t \int_0^1 P_t(f) Y_t(f) df. \]
And $\bar{\Gamma}_t$, the profits made by Northern firms, is the sum of profits made by domestic producers and that made by multinationals. Taking the steady-state value of (16), (20), and (21) and substituting into the above expression, we find that the steady-state consumption is given by
\[ \bar{C} = \bar{\omega}^{1-\mu} \bar{Y}^w \bar{\varphi}_N^{\mu-1} - (1 - G(\bar{\varphi})) F. \] (26)
Similarly, the Southern consumption is given by
\[ \bar{C}^* = \bar{\omega}^{*1-\mu} \bar{Y}^w \left( \bar{\varphi}_N^{*\mu-1} + \bar{\varphi}^{*\mu-1} \right). \] (27)

We prove that in the steady-state, there are positive FDI flows and its direction depends on the average productivities of the countries.

**Proposition.** When two countries have different average productivities and are otherwise
identical, in the steady-state, multinational firms originating from the high-productivity country relocate to produce in the low-productivity country.

Proof. See appendix.

As our focus is economic fluctuations over business cycles of normal magnitude, in later approximations around the steady-state, we assume that the shocks are never big enough to reverse the direction of FDI flow.

4 FDI and Exchange Rate over the Business Cycle

In this section, we examine the dynamics of the model in a first order approximation. In the first subsection, we analytically solve the first-order dynamics of firms' location decisions under a special assumption to gain insights into the model. In the second subsection, we present results from a numerical analysis of the general model.

4.1 First order dynamics at zero fixed cost

An analytically tractable solution to the cross-country distribution of firms is possible when we approximate the model around a steady-state where the fixed investment cost is zero. As we will see, despite being a special case, the analytical study in this section unveils the basic mechanism behind the first-order dynamics of the location distribution of firms.

As PPP holds for prices of consumption baskets in this model, we take the relative labor cost $\omega_t/\omega_t^*$ as the relevant real exchange rate. We first show how are the relative real wage and FDI flows are related. We also look at how nominal exchange rate and FDI flows are related.

Following Melitz (2003) and subsequent literature, we take the fixed cost $F$ as the embodiment of barrier to cross-border FDI flows. When $F$ goes to infinity, it becomes excessively expensive to relocate overseas and cross-border direct investment shuts down, leaving the economy in a domestic production equilibrium. When $F$ shrinks to zero, the economy has no barrier to FDI flows.

As the break-even condition makes clear, the fixed cost also serves as a filter that selects the most productive firms into overseas production. When a finite fixed cost exists, a firm has to be productive enough to earn an expected excess profit at least as large as the fixed cost to move to a location overseas. Thus the set of multinational firms come from the upper truncation of the productivity distribution. When there is no fixed cost at all, we don't exactly know who among the Northern firms become multinationals. But here we consider
the special case $F = 0$ to be the limit of the series of equilibria when $F$ shrinks toward zero. Thus we impose the requirement that the most productive firms are the first to become MNEs.

Denote the log deviation from steady-state of a variable $x$ by $\hat{x} \equiv \log (x/\bar{x})$. The Euler equation can be linearized to

$$
\ddot{Z}_t = \sigma \hat{C}_t + \hat{P}_t - \sigma E_t \hat{C}_{t+1} - E_t \hat{P}_{t+1}.
$$

(28)

As is well known, the combination of complete market and PPP implies that consumption ratio $C_t/C^*_t$ is constant at the initial level. Subtracting the linearized Southern Euler equation from its Northern counterpart, and using $\hat{C}_t = \hat{C}^*_t$ and PPP, we get the uncovered interest parity condition

$$
E_t \hat{S}_{t+1} - \hat{S}_t = i_t - i_t^*.
$$

Following Galí (2008), we define the nominal interest rate as $i_t \equiv -\log Z_t$. Using the interest rate rule (4), we solve the exchange rate depreciation as a function of the monetary shocks

$$
\Delta \hat{S}_t = -\frac{1}{\rho} (\nu_t - \nu_t^*),
$$

(29)

where $\Delta x_t \equiv x_t - x_{t-1}$ denotes the change of variable $x$ from period $t - 1$ to $t$. The nominal exchange rate follows a random walk, and a relative Northern monetary tightening causes a Northern appreciation.

Using the optimal wage-setting rule (7), together with the individual labor demand (2) and aggregate wage evolution (8), we get an equation governing the evolution of the aggregate real wage

$$
(\hat{\omega}_t - \hat{\omega}_{t-1}) = \kappa (\text{MRS}_t - \hat{\omega}_t) + \beta E_t (\hat{\omega}_{t+1} - \hat{\omega}_t) + \beta E_t \pi_{t+1} - \pi_t
$$

(30)

where $\text{MRS}_t \equiv \phi \hat{L}_t + \sigma \hat{C}_t$ is the marginal rate of substitution (MRS) between consumption and leisure, and

$$
\kappa = \frac{(1 - \beta \theta)(1 - \theta)}{\theta \left(1 + \phi \left(\frac{1+\eta}{\eta}\right)\right)}
$$

captures the responsiveness of the aggregate wage to the current period deviation of MRS from the real wage. Subtracting the Southern counterpart of the wage-setting equation (30)

\footnote{The derivation follows closely that of Erceg et al. (2000). See the appendix of that paper for the procedure.}
from the Northern one, and rearranging, we get

\[(1 + \beta + \kappa)\hat{\omega}_t^d = \kappa \phi \hat{L}_t^d + \hat{\omega}_{t-1}^d + \beta E_t \hat{\omega}_{t+1}^d + \beta E_t \pi_{t+1}^d - \pi_t^d, \quad (31)\]

where \(\hat{x}_t^d \equiv \hat{x}_t - \hat{x}_t^*\) denotes the North-South difference in variable \(x\).

From linearizing the aggregate labor demand equations (16) and (18), we get

\[\hat{L}_t^d = -\mu \hat{\omega}_t^d + \hat{\varphi}_t^r,\]

where

\[\varphi_t^r \equiv \frac{\varphi_{N,t-1}^{\mu-1} A_t^{\mu-1}}{\varphi_{N,t-1}^{\mu-1} A_t^{\mu-1} + \varphi_{N,t-1}^{\mu-1} A_t^{\mu-1}}\]

denotes the relative aggregate productivity of firms producing in the two countries. Its first order expansion is

\[\hat{\varphi}_t^r = (\mu - 1) \left[ \hat{\varphi}_{N,t-1} - \left( \frac{\varphi_{N,t-1}^{\mu-1}}{\varphi_{N,t-1}^{\mu-1} + \varphi_{N,t-1}^{\mu-1}} \right) \hat{\varphi}_{N,t-1}^* \right] + \frac{(\mu - 1) \varphi_{N,t-1}^{\mu-1} A_t^d}{\varphi_{N,t-1}^{\mu-1} + \varphi_{N,t-1}^{\mu-1}}.\]

Given the assumption that the productivity distributions are Pareto, the aggregate productivities (17) and (19) can be expressed as functions of the cutoff productivity \(\hat{\varphi}_{t-1}\). We can write the relative labor demand as a function of the real wage gap, cutoff productivity, and relative aggregate productivity

\[\hat{L}_t^d = -\mu \hat{\omega}_t^d + \frac{z}{\kappa \phi} \hat{\varphi}_{t-1} + \frac{(\mu-1) \varphi_{N,t-1}^{\mu-1}}{\varphi_{N,t-1}^{\mu-1} + \varphi_{N,t-1}^{\mu-1}} A_t^d, \quad (32)\]

where

\[z = \kappa \phi \alpha \varphi^{\mu-1-\alpha} \left( \frac{1}{\varphi_{N}^{\mu-1}} + \frac{1}{\varphi_{N}^{\mu-1} + \varphi_{N}^{(\mu-1)}} \right).\]

The relative labor demand depends positively on the cutoff productivity. A higher \(\hat{\varphi}_{t-1}\) means less Northern firms are producing as MNEs in the South, increasing the labor demand in the North relative to South.

When the fixed cost is set to zero, the break-even condition (15) can be linearized to a simple relation

\[E_t(\hat{\omega}_{t+1}^d) = 0. \quad (33)\]

This states that when there’s no barrier to FDI investment, firms will relocate until the real wage rates in the two countries are equalized in expectation.

PPP implies that the difference in inflation rates must be reflected in the nominal ex-
change rate depreciation
\[ \pi_t^d = \Delta \hat{S}_t. \]

We can use the solution to the nominal exchange rate depreciation (29) to substitute out \( \pi_t^d \) and \( \pi_{t+1}^d \) in (31). Substituting also the relative labor demand (32) and the break-even condition (33) into (31), we obtain an expression of the real exchange rate
\[ \hat{\omega}_t^d = \alpha_1 \hat{\varphi}_{t-1}^d + \alpha_2 \hat{\omega}_{t-1}^d + \alpha_3 \nu_t^d + \alpha_4 \hat{A}_t^d, \tag{34} \]

where
\[ \alpha_1 = \frac{z}{(1 + \kappa + \beta + \kappa \phi \mu)} > 0, \]
\[ \alpha_2 = \frac{1}{(1 + \kappa + \beta + \kappa \phi \mu)} \in (0, 1), \]
\[ \alpha_3 = \frac{1}{\rho (1 + \kappa + \beta + \kappa \phi \mu)} \in (0, 1), \]
and
\[ \alpha_4 = \frac{\kappa \phi (\mu - 1)}{(1 + \kappa + \beta + \kappa \phi \mu)} \left( \frac{\varphi^{* (\mu - 1)}}{\hat{\varphi}_N^{\mu - 1} + \varphi^{* (\mu - 1)}} \right) > 0. \]

These coefficients are all positive. The equilibrium relative real wage depends positively on the lagged cutoff productivity, because higher cutoff productivity means more Northern firms are producing at home, increasing the relative labor demand in the North. The sluggish wage adjustment means the relative real wage also depends positively on its own lagged value and the monetary shocks. A relative monetary tightening in the North causes real wage rate in the North to rise relative to the South. Finally, a Northern productivity gain causes the real wage to rise in the North through increased labor demand.

We substitute (34) into the linearized break-even condition (33) to get a solution to the cutoff productivity
\[ \hat{\varphi}_t = -\alpha_2 \hat{\varphi}_{t-1}^d - \frac{\alpha_2 \alpha_3}{\alpha_1} \hat{\omega}_{t-1}^d - \left( \frac{\alpha_2 \alpha_4}{\alpha_1} \nu_t^d + \zeta \frac{\alpha_4}{\alpha_1} \right) \hat{A}_t^d. \tag{35} \]

The fraction of Northern firms who will be producing as multinationals in period \( t + 1 \) is \( 1 - G(\hat{\varphi}_t) \). Thus the cutoff productivity is negatively related to the size of FDI. The solutions to the relative real wage and the cutoff productivity have made it clear that the real exchange rate, as measured by the relative real wage, and the FDI inflow in a given period are positively correlated, under either nominal shocks or productivity shocks.

The break-even condition for the marginal multinational firm is the key to understand this correlation. Essentially, the break-even condition states that firm relocation will continue
until the expected North-South real wage gap shrinks to a level at which the marginal
firm cannot expect to make a profit anymore. (In the special case $F = 0$, the real wages
have to equalize in expectation.) The correlation in the case of nominal shocks is relatively
straightforward. A relative Northern monetary tightening increases the relative Northern
real wage, as under the sluggish wage-setting, the nominal wages are not responsive enough
to offset the appreciation of the nominal exchange rate. Also as a consequence of the sticky
wage, the relative real wage is history-dependent. Thus the current wage gap is expected
to persist into the next period. This induces more Northern firms to commit to producing
abroad in the next period, causing an increased FDI flow into the South.

A relative Northern productivity gain drives up the relative real wage, as the average firm
producing in the North is more productive. Firm relocation increases for two reasons. First,
productivity processes are persistent. The higher Northern productivity today is expected
to be inherited next period. Thus the real wage gap is expected to continue existing, causing
more firms to relocate. Second, the sticky wage-setting still implies that the current wage gap
is going to continue into the next period, again causing more MNE relocation. The relocation
caused by the persistence of productivity will happen with or without sticky wage, and is
captured in equation (35) by the last term $\left( \frac{\alpha_4}{\alpha_1} \right) \hat{A}_t$. It is easy to verify that this term
represents the efficient firm relocation under flexible wages. Note that the existence of sticky
wage tends to “strengthen” the FDI flow that would happen under flexible wages, so that
firm relocation overshoots the efficient level in the period when the productivity shock hits.

Turning to the nominal exchange rate, with a relative Northern monetary tightening
($\nu_t > 0$), the North (South) experiences an appreciation (a depreciation) together with
increased FDI outflow (inflow). Shocks that increase the relative productivity $\hat{A}_t$ cause
increased FDI flow from North to South, but leave the nominal exchange rate unchanged.
While the nominal exchange rate is clearly positively correlated with FDI inflow overall,
it should be noted that the unresponsiveness of the nominal exchange rate to productivity
shocks is specific to the model assumptions. For example, if we allow monetary policy to
follow a general Taylor rule, so that it targets the output gap in addition to inflation, nominal
exchange rate would appreciate in response to a positive productivity shock, enhancing the
positive correlation between the nominal exchange rate and FDI inflows.

4.2 Numerical simulation

In this subsection, we numerically simulate the model for the general case when the fixed
cost is positive, and highlight several key features.

The parameterization mainly follows Erceg et al. (2000). We set a (quarterly) discount
rate $\beta = 0.99$, and utility parameters $\sigma = \phi = 1.5$. We choose an elasticity of substitution $\mu = 4$, and wage mark-up rate $\eta = 1/3$. The probability of wage stickiness is set to $\theta = 0.75$. This corresponds to an average wage contract duration of one year. Following Russ (2007), we set $\alpha = \mu + 0.1$ to make the dispersion of firm productivity close to one, as the cross-industry estimation by Helpman et al. (2004) suggests. The Southern minimum productivity is set to $1/2$. Next, we set the fixed cost to 0.01. In the steady-state, about 7% Northern firms produce as multinationals. Regarding the interest rate rule, the inflation response coefficient is set to $\rho = 1.5$. The persistence coefficient of the aggregate productivity is set to $\zeta = 0.9$. Finally, the monetary and productivity shocks have a standard deviation of 0.02.

Figure 1 plots the impulse response functions (IRFs) of a set of variables of interest to a Southern monetary expansion (that is, a one standard deviation drop in $\nu^*_t$). The monetary expansion causes inflation and drives down real wage in the South. As Northern currency appreciates against the South, prices of imported goods drop and drag down the aggregate price level, causing an increase in the Northern real wage. The enlarged real wage gap induces more Northern firms to relocate production to the South, causing the mass of MNE to increase.

Figure 2, on the other hand, plots IRFs to an increase in Northern productivity (a one standard deviation shock to $\varepsilon_t$). Higher productivity lowers marginal cost, putting downward pressure on prices of goods produced by Northern firms. Deflation drives up real wages in both countries. Meanwhile, at any given real wage gap, Northern firms find it more profitable to relocate to the South, as their productivity is higher, increasing the expected gain from relocation. The IRFs show that the mass of MNEs moves positively with the relative real wage period-by-period. Specifically, in the second period after the shock, the relocation of MNEs to the South puts so much upward pressure on the labor demand in the South, that it drives the relative real wage down to a level slightly below the steady-state. This happens because the productivity gain by Northern firms is large enough, so that relocation is profitable even if the real wage gap is smaller. When the relative real wage recovers again in the following period, so does the mass of MNEs.

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13Helpman et al. (2004) estimated productivity dispersion, measured by the value of the coefficient $\alpha - (\mu - 1)$, using industry level data for U.S. and European firms. The estimations of these industry-specific dispersion parameters range from 0.52 to 2.97. The qualitative simulation results in this section are robust to these alternative dispersion parameters. See Helpman et al. (2003), Table A.1.

14We try different parameter values to make sure the qualitative results are robust to alternative parameterizations.
Discussion: The cyclicality of real wage

Changes in relative production costs - the relative real wage - is a main driver of MNE relocations in this model. Given its importance, it is worthwhile to examine the empirical fitness of the cyclical behavior of real wage. The model predicts that the aggregate real wage is pro-cyclical under productivity shocks, and counter-cyclical under monetary shocks. In the benchmark simulation, the aggregate real wage of each country is moderately positively correlated with its output. But what the model really predicts is that if an economy is hit by both productivity and monetary shocks, the aggregate real wage could either be procyclical or countercyclical, depending on the relative volatility of the shocks. This is consistent with the empirical evidence on the acyclicality of aggregate real wage in the US (Abraham and Haltiwanger (1995)). It is also consistent with the cross-country heterogeneity in real wage cyclicality at the aggregate level (Messina et al. (2009)).

Recent advances in empirical labor economics were largely guided by the search and matching model. New empirical works typically use matched micro-level data, which allows a careful examination of subgroups of the labor force. A consensus is that the wage rate of new matches is procyclical (Pissarides (2009)). This is also consistent with the model
The prediction of the model seems to be consistent with the view that a country can attract more FDI inflow by engineering a depreciation, but it also predicts that the outcome is undesirable from the depreciating country’s point of view.

Going back to Figure 1, where the economy experiences an unexpected Southern monetary expansion, the Southern real wage drops. The more productive Northern firms are attracted by the lower real wage abroad after the shock, and their relocation means the demand for Southern labor is further increased. Note that without these new MNEs, Southern (Northern) workers would already be working too much (too little) compared to the flexible-price long-run equilibrium level, as the Southern monetary expansion lowers (raises) the Southern (Northern) real wage. The entry of new MNEs thus drives the aggregate employments in both countries further away from their long-run equilibrium level.

Figure 3 shows the IRFs of outputs and period utilities after the Southern monetary expansion. Southern output increases sharply as Southern employment expands. Northern output drops. The welfare effects of these cross-border relocations after a monetary shock is asymmetric across the two countries. As the Southern real wage depreciation attracts
more FDI inflow, households are worse off as they end up working more, while households in the North enjoy more leisure. Thus the model suggests that an unanticipated depreciation attracts more FDI inflow, but leaves the domestic agents worse off as they work more under sticky wage contracts. It is not beneficial to the domestic household to manipulate the exchange rate to attract FDI inflow. Rather, the reversed operation of an artificial appreciation that drives firms abroad is welfare-improving. Of course, the gain is only to the appreciating country, the world as a whole loses utility from such deviations caused by nominal rigidity.

The fact that firm relocation exacerbates employment deviations under nominal shocks is consistent with the empirical finding of Bergin et al. (2009). In Figure 3 we examine how period utility changes after the shock. The interesting question is, if relocation of MNEs exacerbate the business cycle fluctuations around the efficient steady-state level, what are the

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15 Bergin et al. (2009) emphasize that the Mexican *maquiladora* industry has higher employment volatility than its US counterpart. In our model though, we stress that the effect is bilateral. Of course, if the model is revised so that the South is much smaller than the North, we would expect to see the same firm relocations to create a larger impact on the South.
welfare implications of multinational production as an institution? We turn to this question in the next section.

5 Welfare Effects of FDI

In this section we numerically simulate the model with second order approximation to examine how multinational production affects world welfare. We answer the question by examining the model in environments with nominal shocks and real shocks separately. Our model with firm relocation (henceforth referred to as the “benchmark model”) is compared with two alternative models. The domestic production model is the benchmark model without cross-border firm relocation. It can be thought of as the limit of the benchmark model as the fixed cost $F$ goes to infinity. The steady-state MNE model (henceforth “ssMNE model”), on the other hand, is generated by imposing the invariant steady-state MNE distribution on the benchmark model. That is to say, all Northern firms who are MNEs in the steady-state are required to stay producing in the South, and no more moving back-and-forth is allowed over the business cycle. In this way, we can hypothetically separate the steady-state relocations of the long term nature from the marginal relocations that only happen over the business cycle. Consistent with previous literature, long-run FDI is beneficial. The welfare effect of firm relocation over the business cycle, however, differ under nominal shocks and real shocks. Under nominal shocks, the marginal MNE relocation decreases welfare by magnifying business cycle fluctuations. Under real shocks, it has an ambiguous welfare effect. In terms of magnitude, we show that the welfare impact of FDI under nominal shocks is much larger than that under real shocks. We conclude that in an environment with mixed nominal and real shocks, the model that allows only long-term firm relocation produces the highest level of welfare.

5.1 Welfare implication of firm relocation under monetary shocks

In this subsection, we shut down real shocks, and examine the welfare effects of FDI under nominal shocks only. The world welfare is defined as the unconditional expectation of the unweighted sum of Northern and Southern household average period utilities, that is,

$$W_t = U(C_t) - \int_0^1 V(N_t(h))dh + U(C^*_t) - \int_0^1 V(N^*_t(h^*))dh^*.$$

We compare the expected world welfare in the benchmark model with that in the domestic production model and the ssMNE model. To compute the expected world utility, we simulate
the model for two thousands periods, and drop the first two hundred. We repeat this for various values of monetary volatilities, measured by the standard deviations of the shocks to interest rates \( \nu \) and \( \nu^* \). We compute the expected utility under these different values of standard deviations, ranging from 0 to 0.1, at an increment of 0.001.

Figure 4 reports the results of these simulations. The solid blue curve depicts the world utility computed from the benchmark model, when all firms can freely relocate cross-border subject to the fixed cost \( F \). The dotted green curve is the world utility under domestic production, i.e., when all firms are restricted to produce in their domestic country. Clearly, the long-run FDI flows that takes place in the steady-state are welfare-improving. This steady-state welfare improvement is consistent with insights from previous literature on the long-run gain of FDI. The North gains from access to Southern labor which lowers the cost of production. The South gains by accessing superior technology from the North.

![Figure 4: World welfare for different levels of monetary shock volatility](image)

However, while welfare under both the FDI and domestic production scenarios decrease with the volatility of monetary shocks, the rate of decrease is higher with FDI flows. An inspection of terms that affect welfare show that FDI’s impact on aggregate employment is the culprit. When the volatility of monetary shocks gets higher, on average, the relative
real wage is driven further away from its flexible-price equilibrium level. This then induces more temporary relocation of firms, causing aggregate employments to deviate even more, as we have shown in the last section. Thus the option to relocate in the short run amplifies the inefficiency caused by the nominal rigidity in wage-setting. As a result, if monetary volatility is sufficiently high, it becomes welfare-improving to shut down cross-border FDI flows altogether. In our baseline calibration, the utility with FDI drops below the utility under domestic production at a standard deviation of 4.3%.

These welfare results under nominal shocks can be thought of as an application of the theory of second best. When a friction causes a market failure, allowing market participants to freely exploit the opportunities caused by the friction could magnify the overall welfare loss. In our specific case, the same FDI flow induced by a lower production cost abroad could be beneficial or harmful depending on the underlying cause of that lower real wage in the first place. It is welfare-improving when the real wage gap is caused by the fundamental difference in productivity, but exacerbates the welfare loss when the wage gap is caused by nominal frictions.

Therefore, there should be no surprise that the best outcome occurs under the scenario that only keeps the MNE relocations of the long-run nature. The top dotted red curve with circular markers in Figure 4 depicts the world welfare when we simply keep the steady-state distribution of firms, but disallow further moving back-and-forth. World utility is highest under this scenario at every monetary volatility level, as the economy is able to reap the gain from long-run firm relocation, but avoid the harm caused by short-run relocations aimed to take advantage of nominal rigidities.

5.2 Welfare implication of firm relocation under productivity shocks

This subsection looks at the welfare effects of FDI in an environment with productivity shocks. We again compare world welfare under the benchmark model, the domestic production model, and the ssMNE model.

Figure 5 depicts the welfare in the three models when the standard deviation of productivity shocks increases from zero to 0.1. The solid blue curve is welfare under the benchmark model, the dotted green curve is welfare under the domestic production model, and the thin red curve with circular markers is the welfare under the ssMNE model.

Clearly, both the benchmark and ssMNE models dominate the domestic production model. In the relevant range of productivity shock volatility, the loss from the absence of FDI mainly comes from the failure to capture the long-run gain of FDI.
A comparison of the welfare under the benchmark model and the ssMNE model reveals that with real shocks only, the welfare effect of the marginal MNE relocation over the business cycle is negligible. Graphically, the two curves representing welfare under the benchmark model and the ssMNE model almost overlap.

A more careful examination indicates that the proximity of the welfare under the two models is a result of counteracting forces. The marginal MNE relocation benefits the economy in two ways. It increases mean consumption in both countries, as the relocation motivated by productivity changes allows MNEs to reduce production cost. MNE relocations also serve as an “employment risk-sharing” mechanism between the two countries. Consider a positive productivity shock in the North. Absent firm relocations, the Northern productivity gain makes domestic workers work harder. Meanwhile, Southern consumers enjoy more leisure because Southern consumption increases under a complete asset market, driving down the marginal utility gain of working. Relocations of MNEs “redistribute” labor efforts in an equalizing way. With the Northern productivity gain, more Northern firms find it profitable to move abroad, increasing labor demand in the South, while reducing it in the North. As the utility function is concave in leisure, this redistribution improves world welfare, in the same way as the redistribution of consumptions by complete markets. Figure 6 in the appendix illustrates this redistribution effect.

Figure 5: World welfare for different levels of productivity shock volatility
With these benefits, why wouldn’t the marginal MNE relocations be welfare-improving? The answer lies in nominal rigidity. With sticky wages, MNE distribution deviates from the efficient flexible-wage equilibrium. When a positive productivity shock hits the North, for example, Northern firms “over-relocate” upon impact. We have analytically shown this mechanism in Section 4.1, after deriving the cutoff productivity. The over-relocation drives labor efforts away from the efficient level and causes a loss of welfare.

The resulting welfare implication then reflects a tradeoff between the gain from consumption increase and labor effort sharing, and the loss from the distortion. The gain is available under flexible or sticky wages, but the loss is only a consequence of nominal rigidity. This suggests that the net welfare gain from the relocation of marginal MNEs increases with the degree of nominal wage flexibility. Indeed, when we compute world welfare under flexible wages, which can be considered as the limiting case of the benchmark model when the probability of resetting wage goes to one, the benchmark model yields higher utility at every level of real shock volatility.

However, note that under productivity shocks, the difference in the magnitudes of welfare produced by the benchmark model and the ssMNE model is extremely small. Even when wage is completely flexible, so that the loss from nominal distortions is eliminated, and the standard deviation of the productivity shocks reaches 0.1, the gain in world welfare from having marginal MNE relocations is still so small that agents in the model are only willing to give up 0.14% of steady-state consumption to go from the ssMNE model to the benchmark model. In contrast, in the benchmark calibration, when the standard deviation of nominal shocks is 0.02, agents are willing to give up 3.24% of steady-state consumption to shut down the marginal relocations.

The difference in the magnitude of the welfare implications thus suggests that, at business cycle frequency, the welfare effect of short-run FDI flows mainly comes from its magnification of fluctuations caused by nominal shocks. In an environment when the source of economic fluctuations is unobservable, the ssMNE model in which the marginal MNE relocation is disabled produces the highest welfare level for any reasonable mixture of monetary and productivity shocks.

6 Conclusion

This paper provides an explanation of the positive correlation between exchange rate and FDI inflow, and examines the short-run welfare effects of FDI flows in an open-economy framework.
macro model. We analyze firms’ cross-border production location decisions in a model in which both the exchange rate and FDI flow are endogenously determined. The paper shows that FDI flow and exchange rate are positively correlated under monetary or productivity shocks. Welfare analysis shows that short-run FDI fluctuations exacerbate utility loss in an environment with nominal shocks, but has little impact on welfare over business cycles caused by productivity shocks. Thus the best outcome can be achieved if long-term FDI is retained, but short-run movement in the production location is disallowed.

An interesting question is then, how can a policy be implemented to discourage the short-run firm relocations when the steady-state distribution is unknown, or when firm productivity is not directly observable? One possibility is to require a minimum operation time. For example, a country could disallow sale of principal capital for several years. A country might also impose a capital tax to discourage the temporary relocations. In either case, the key is to make entry profitable for the long-term investors, but deter the short-term profit seekers. Designing such a policy would be an interesting avenue for future research.

7 Appendix

7.1 FDI Flows

**Proposition.** When two countries have different average productivities and are otherwise identical, in the steady-state, multinational firms originating from the high-productivity country relocate to produce in the low-productivity country.

**Proof.** We prove by contradiction.

Case 1: Assume there is no cross-border firm relocation in the steady-state.

This means that even the most productive firm cannot gain from relocation. The break even condition (15) implies that real wages must equalize. Thus (25) becomes

\[
\frac{\bar{C}}{\bar{C}^*} = \left( \frac{\varphi^{\mu-1}}{\varphi^{\mu}-1} \right)^{-\frac{1}{\mu-1}} \tag{36}
\]

where

\[
\varphi \equiv \left[ \int_1^{\infty} \varphi^{-1} g(\varphi) d\varphi \right]^{1/\mu-1}
\]

is the average productivity of Northern firms. By assumption, \( \varphi > \varphi^* \), and therefore (36) implies \( \bar{C}/\bar{C}^* < 1 \). On the other hand, without foreign production, each country gets to consume its domestic output. Set \( \varphi_H^{*\mu-1} = 0, \varphi_H = \varphi \) and \( G(\varphi) = 1 \) in the two budget
constraints (26) and (27), and then divide to get

\[
\frac{\bar{C}}{\bar{C}^*} = \frac{\bar{\varphi}^{\mu-1}}{\bar{\varphi}_{S}^{\mu-1}} > 1,
\]

which leads to a contradiction with the prediction from the labor market equilibrium condition.

Case 2: Assume that in steady-state, FDI flows from South to North.

If this is true, the breakeven condition (15) implies that the real wage must be higher in the South, i.e. \( \bar{\omega}^* > \bar{\omega} \). We have the counterpart of (25), which can be written as

\[
\left( \frac{\bar{C}}{\bar{C}^*} \right)^{\frac{\bar{\omega}}{\bar{\omega}^*}} = \left( \frac{\bar{\omega}}{\bar{\omega}^*} \right)^{\frac{\bar{\varphi}_{S}^{\mu-1}}{\bar{\varphi}_{S}^{\mu-1} + \bar{\varphi}^{\mu-1}}}.
\]

(37)

where

\[
\bar{\varphi}_{S}^* \equiv \left[ \int_{\bar{\varphi}_{m}}^{\bar{\varphi}^*} \varphi^{\mu-1} g(\varphi) d\varphi \right]^{\frac{1}{\mu-1}}
\]

and

\[
\bar{\varphi}_{S} \equiv \left[ \int_{\bar{\varphi}}^{\infty} \varphi^{\mu-1} g(\varphi) d\varphi \right]^{\frac{1}{\mu-1}}
\]

are the aggregate productivities of Southern firms who produce domestically and abroad respectively, and \( \bar{\varphi}^* \) is the cutoff productivity of the marginal Southern MNE. Since \( \bar{\varphi}^{\mu-1} > \bar{\varphi}_{S}^{\mu-1} > \bar{\varphi}_{S}^{\mu-1} \), and \( \bar{\omega} > \bar{\omega}^* \), (37) implies that \( \bar{C}/\bar{C}^* < 1 \).

By symmetry, the two budget constraints (26) and (27) imply

\[
\bar{C} = \bar{\omega}^{1-\mu} \bar{Y}^w (\bar{\varphi}_{S}^{\mu-1} + \bar{\varphi}^{\mu-1})
\]

and

\[
\bar{C}^* = \bar{\omega}^*^{1-\mu} \bar{Y}^w \bar{\varphi}_{S}^{\mu-1} - (1 - G^* (\bar{\varphi}^*)) F.
\]

The labor market equilibrium states that \( \bar{C}/\bar{C}^* < 1 \), and the last term is a non-negative number. Therefore we must have

\[
\bar{\omega}^{1-\mu} \bar{Y}^w (\bar{\varphi}_{S}^{\mu-1} + \bar{\varphi}^{\mu-1}) < \bar{\omega}^*^{1-\mu} \bar{Y}^w \bar{\varphi}_{S}^{\mu-1},
\]

which implies

\[
\left( \frac{\bar{\omega}}{\bar{\omega}^*} \right)^{1-\mu} < \frac{\bar{\varphi}_{S}^{\mu-1}}{\bar{\varphi}_{S}^{\mu-1} + \bar{\varphi}^{\mu-1}} < 1,
\]

(38)

as the North has higher average productivity. But the previous inequality (38) also implies
that $\bar{\omega}^* < \bar{\omega}$, which leads to a contradiction, as the break-even condition states that when there is positive FDI flow from South to North, Southern real wage must be higher than the Northern real wage. Therefore Case 2 cannot be true. The only possible equilibrium is for multinational firms to relocate from the country with high productivity to the country with low productivity.

7.2 Welfare under productivity shocks

Figure 6 depicts IRFs of Northern and Southern employments in response to a positive Northern productivity shock. Outer lines with cross markers are IRFs produced by the ssMNE model, and inner lines are IRFs produced by the benchmark model. Blue lines are IRFs of Northern employment, and green lines are IRFs of Southern employment. Compared with the ssMNE model, the marginal MNE relocation that takes place over the business cycle tends to redistribute employment effort in an equalizing way.

![Figure 6: IRFs of employment to a shock to Northern productivity](image)

Figure 6: IRFs of employment to a shock to Northern productivity

Figure 7 shows IRFs of the mass of MNEs under sticky wages and flexible wages, respectively. The solid blue line depicts the mass of MNEs for a sticky wage rate while the
The dashed green curve depicts the mass of MNEs for a flexible wage rate. Response under the frictionless model is efficient. Thus the deviation of the solid blue curve from the dashed green curve represents distortions caused by sticky wage contracts.

![Diagram](image_url)

**Figure 7:** IRFs of the mass of MNEs to a shock to Northern productivity

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