A Model of Trade with Ricardian Comparative Advantage and Intra-sectoral Firm Heterogeneity*

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Abstract

In this paper, we merge the heterogenous firm trade model of Melitz (2003) with the Ricardian model of Dornbusch, Fisher and Samuelson (DFS 1977) to explain how the pattern of international specialization and trade is determined by the interaction of comparative advantage, economies of scale, country sizes and trade barriers. The model is able to capture the existence of inter-industry trade and intra-industry trade in a single unified framework. It explains how trade openness affects the pattern of international specialization and trade. The paper examines the generality of Melitz's firm selection effect in a multi-sectoral setting. Although opening to trade is unambiguously welfare-improving in both countries, trade liberalization can lead to a counter-Melitz effect in the larger country in the sectors where it has the strongest comparative disadvantage yet still produces. In this case, the productivity cutoff for survival is lowered while the exporting cutoff increases in the face of trade liberalization. This is because the inter-sectoral resource allocation (IRA) effect dominates the Melitz effect in these sectors. Consequently, the larger country can lose from trade liberalization. Some hypotheses related to firms’ exporting behavior across sectors upon trade liberalization are also derived. Analyses of data of Chinese manufacturing sectors confirm these hypotheses.

Keywords: inter-industry trade, intra-industry trade, heterogeneous firms, trade liberalization

JEL Classification codes: F12, F14

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1 Introduction

How do firms’ entry, exit, output and exporting decisions respond to trade integration and trade liberalization? Do they respond differently across sectors? How is trade pattern determined by the interaction between pattern of comparative advantage across sectors and monopolistic competition between firms within the same sector? How are trade pattern and welfare affected by globalization? We try to answer these questions by developing a model of trade with comparative advantage across sectors and intra-sectoral firm heterogeneity.

There are by and large two types of international trade: inter-industry trade and intra-industry trade. It is widely recognized that the former is driven by comparative advantage and the latter by economies of scale. The most widely used models for capturing comparative advantage are of the Ricardian type (e.g. Dornbusch, Fisher and Samuelson (DFS) 1977, Eaton and Kortum 2002) and the Heckscher-Ohlin type. The most notable models used to capture intra-industry trade are probably attributed to Krugman (1979, 1980). More recently, Melitz (2003) extends Krugman’s (1980) model to analyze intra-industry trade when there is firm heterogeneity, thus capturing the selection of firms according to productivity and profit-shifting to firms of higher productivity when a country opens up to trade and trade liberalization. It stimulates much further work in this direction, notably Chaney (2008), Melitz and Ottaviano (2008), and Arkolakis (2010), to name just a few papers.

Most papers analyzing trading economies focus their analysis on the effects attributed to one single trade model. For example, they assume that the world is described by an Armington, Krugman, DFS, Eaton and Kortum, or Melitz model. Thus, they ignore the interaction of the various effects when both comparative advantage and economies of scale are present so that there are both inter-industry and intra-industry trade between countries. This paper proposes a unified framework to capture both inter-industry and intra-industry trade with firm heterogeneity in a single model. By doing so, we have a model that explains how comparative advantage, economies of scale, firm selection and home market effect interact to sort sectors into ones in which only one of the countries produces (where there is inter-industry trade) and ones in which both countries produce (where there is intra-industry trade). Specifically, we modify Dornbusch, Fischer, and Samuelson’s (1977) two-country, multi-sector Ricardian framework by incorporating intra-sectoral firm heterogeneity a la Melitz (2003). A number of testable hypotheses are generated. For example, sectors in which one of the countries has strong comparative advantage would be characterized by inter-industry trade, while sectors in which neither country has strong comparative advantage would be characterized by intra-industry trade. For any given country, the fraction of firms that export is higher for a sector with stronger comparative advantage. These results are supported by Chinese data.

Furthermore, we are able to understand the interaction of the forces attributed to comparative advantage effect, productivity selection effect and home market effect, in the face of trade integration and trade liberalization. We find that we can always decompose the total effect of trade liberalization into those caused by inter-sectoral resource allocation (IRA) effect and firm selection effect according to productivity (which we call Melitz effect).
Although trade integration (switching from autarky to trade) is always welfare-improving, the welfare effect of trade liberalization (reduction of trade barriers) depends on the relative size of the two countries, the height of trade barriers and the Ricardian technological differences between the two countries. In particular, in the case of trade liberalization, we find that the interaction of the IRA effect, the Melitz effect and home market effect in certain sectors can give rise to an effect that is opposite to what is predicted by Melitz. For lack of a better term, we call this effect counter-Melitz effect. Melitz predicts that trade liberalization leads to an increase in the productivity cutoff for survival but a decrease in the exporting productivity cutoff, and this gives rise to an increase in the average productivity of the firms that serve the domestic market, leading to an increase in domestic real wage in terms of goods in the sector. The counter-Melitz effect in our model predicts that, in some sectors, trade liberalization leads to a decrease in the productivity cutoff for survival but an increase in the exporting productivity cutoff, and this gives rise to a decrease in the average productivity of the firms that serve the domestic market, leading to domestic real wage reductions in terms of goods in these sectors. This is because the IRA effect dominates the Melitz effect in the sectors where the larger country has the strongest comparative disadvantage and yet still produces. The reason why the larger country can profitably produce in the sectors in which it has comparative disadvantage is the existence of increasing returns to scale and home market effect as explained by Krugman (1980). For this reason, the counter-Melitz effect cannot exist in the smaller country.

Therefore, there exists a counter-Melitz effect in the comparative disadvantage sectors and a Melitz effect in the comparative advantage sectors, if the country is large. This implies that for a given country, in the face of trade liberalization, the fraction of exporters and the share of revenue in total revenue both increase in the comparative advantage sectors but they both decrease in the sectors in which the country has the least comparative advantage yet still produces, if the country is large. The theory is confirmed by testing the hypotheses with Chinese firm-level data.

In the recent theoretical literature modeling open economy with heterogeneous firms, papers by Okubo (2009) and Bernard, Redding and Schott (2007) are the closest to ours. Like us, Okubo (2009) also introduces multiple sectors into the Melitz model, thus making it a hybrid of the multiple-sector Ricardian model and the Melitz model. In the two-sector case he analyzes the general equilibrium effects, allowing the endogenous determination of the relative wage. But the focus of his paper is quite different from ours, though there are some similarities. He mainly focuses on changes in population and the effects on the number of varieties. We mainly focus on how the strength of comparative advantage of a sector affects firm selection in different sectors under trade integration and trade liberalization. We analyze and obtain closed form solution of the international pattern of specialization and trade as a function of trade barriers, relative country size and Ricardian comparative advantage. We decompose the total effect of trade liberalization into the IRA effect and Melitz effect and explain the condition under which one effect can dominate the other. Most importantly, we identify the conditions under which there is a counter-Melitz effect and a loss from trade liberalization in certain sectors or even the entire economy. We carry out empirical tests of the hypotheses while he does not conduct any empirical tests.
Bernard, Redding and Schott (2007) incorporate firm heterogeneity into a two-sector, two-country Heckscher-Ohlin model, and analyze how falling trade costs lead to the reallocation of resources, both within and across industries. Inter-sectoral resource reallocation changes the ex-ante comparative advantage and provides a new source of welfare gains from trade as well as causes redistribution of income across factors. In their paper, trade liberalization raises the productivity cutoff for survival and lowers the exporting productivity cutoff in both industries, with the effect being disproportionately larger in the comparative advantage sectors. Therefore, there is no counter-Melitz effect in their paper.

One could have captured inter-industry trade and intra-industry trade in a unified model without assuming firm heterogeneity.\(^1\) As is found elsewhere in the literature, the aggregate results remain about the same whether or not firm heterogeneity is assumed. Probably the largest benefit from incorporating firm heterogeneity is that we are able to use firm level data to confront some of the hypotheses. For example, some propositions contain predictions about the variation in the percentage of firms that export across sectors. Such propositions cannot be derived from a model with homogeneous firms.

Recently, Lu (2010) found that in sectors where China had comparative advantage (i.e. labor-intensive sectors), Chinese firms that mainly exported were on average less productive than those that served the domestic market. Does her finding contradict our theory or empirical finding here? The answer is no. First, as far as theory is concerned, our model can be easily modified to yield the same theoretical prediction as Lu by making a slight change in the assumption about market-entry costs. This modification is explained in the penultimate section of this paper. Second, even with this modification, all the propositions in our paper remain the same. Most importantly, the model continues to yield the counter-Melitz effect. In other words, our basic model can be modified to capture the peculiar feature of some Chinese exporting firms (namely firms engaging in processing trade) and yield predictions that are consistent with the empirical findings of this paper as well as those of Lu (2010) and her critics (e.g. Dai, Maitra and Yu (2011)).

The paper is organized as follows. Section 2 presents the model with heterogeneous firms in the closed economy and examines the properties of the equilibrium. In section 3, we carry out an analysis of the equilibrium in the open economy. We analyze the pattern of specialization and trade and identify the existence of inter-industry trade as well as intra-industry trade. In section 4, we show the impact of trade integration on the productivity cutoffs, the number of firms and welfare in each sector. In section 5, we analyze the effects of trade liberalization, and demonstrate the existence of a counter-Melitz effect in certain comparative disadvantage sectors of the large country. Empirical tests of the main propositions of the section are carried out. In section 6, we modify our assumption to allow for the possibility that some firms only export but do not serve the domestic market. This modified version of the model yields predictions consistent with our empirical results as well as other recent findings about Chinese exporting behavior, such as Lu (2010) and Dai, Maitra and Yu (2011). Section 7 concludes.

\(^1\)For example, Helpman and Krugman (1985) integrate the inter-industry trade and intra-industry trade model with homogeneous firms.
2 A Closed-economy Model

In this section, we shall describe the features of a closed economy, but where necessary we also touch upon some features of a two-country model when the closed economy opens up to trade. The closed economy is composed of multiple sectors: a homogenous-good sector, and a continuum of sectors of differentiated goods. There is only one factor input called labor. The homogeneous good is produced using a constant returns to scale technology. It is freely traded with zero trade costs when the country is opened up to trade. Firms are free to choose the sectors into which they enter. We assume that in order to produce a differentiated good, a firm has to pay a sunk cost of entry. After entry, a firm decides whether or not to produce according to whether the expected present discounted value of its economic profit is non-negative after its firm-specific productivity has realized. The economic profit is determined by the following factors. There is a fixed cost of production per period, and a constant marginal cost of production. The fixed cost of production is the same for all firms but the marginal cost of production of a firm is partly determined by a random draw from a distribution. Upon payment of the entry cost \( f_e \), the firm earns the opportunity to make a random draw from a distribution of firm productivity. The draw will determine the firm-specific component of the firm’s productivity (i.e. reciprocal of the unit labor requirement for production). The above characterizations of the model are basically drawn from Melitz (2003). Unlike Melitz, there is another factor that affects the variable cost of production of a firm, which is an exogenously determined sector-specific technological level. In general, this technological level differs across sectors in a country as well as differs across countries within the same sector. The set of sector-specific technological levels across sectors in both countries determine the pattern of comparative advantage across sectors of each country. The above structure is basically drawn from DFS (1977). Our model is therefore a hybrid of Melitz (2003) and DFS (1977).

There are \( L \) consumers, each supplying one unit of labor. Preferences are defined by a nested Cobb-Douglas function:

\[
\ln U = \alpha \ln C_h + \int_0^1 b_k \ln C_k dk
\]

\[C_k = \left[ \int_0^{\theta_k} c_k(j)^{\theta} dj \right]^{\frac{1}{\theta}} \text{ with } \int_0^1 b_k dk = 1 - \alpha\]

where \( \alpha \) denotes the share of expenditure on homogenous goods, \( b_k \) is the share of expenditure on differentiated good \( k \in [0, 1] \); \( \theta_k \) is the endogenously determined mass of varieties in differentiated sector \( k \). The homogeneous good is produced with constant unit labor requirement \( 1/A_h \). The price of the homogeneous good is \( w/A_h \), where \( w \) is the wage, as it is produced and sold under perfect competition. For the differentiated-goods sectors, the exact price index for each sector is denoted by \( P_k \), where

\[P_k = \left[ \int_0^{\theta_k} p_k(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}, \text{ where } \sigma = \frac{1}{1-\rho} > 1\]

where \( p_k(j) \) denotes the price of variety \( j \) in sector \( k \), and \( \sigma \) denotes the elasticity of substitution between varieties. Cost minimization by firms implies that the gross revenue flow of firm \( j \) in sector \( k \) is given
by

\[ r_k(j) = b_k E \left[ \frac{p_k(j)}{\bar{p}_k} \right]^{1-\sigma} \]  

(3)

where \( E = wL \) denotes the total expenditure on all goods.

The labor productivity of a firm in the differentiated-good sector \( k \) is the product of two terms: one is firm-specific, being a random variable following a Pareto distribution \( P(1, \gamma) = 1 - \left( \frac{1}{\varphi} \right)^\gamma \) where \( \varphi \in [1, \infty] \) and \( \gamma \ (\sigma - 1) \) is the shape parameter of the distribution; the other is \( A_k \), which is exogenous and sector-specific. The labor productivity of a firm is thus equal to \( A_k \varphi \). Production labor employed by firm \( j \) in sector \( k \) is a linear function of output \( y_k(j) \):

\[ l_k(j) = f + \frac{y_k(j)}{A_k \varphi_k(j)}, \]

where \( f \) is the fixed cost of production per period, and \( A_k \varphi_k(j) \) is the productivity of firm \( j \) in sector \( k \), . Therefore, under monopolistic competition in sector \( k \) the profit-maximizing price is given by

\[ p_k(j) = \frac{w}{\rho A_k \varphi_k(j)} \]  

(4)

(3) and (4) imply that the flow of profit of the firm is given by

\[ \pi_k(j) = \frac{r_k(j)}{\sigma} - f w = \frac{b_k w L \left[ P_k w^{-1} p A_k \varphi_k(j) \right]^{\sigma-1}}{\sigma} - f w \]

If a firm draws too low a productivity, it will exit immediately, as the expected present discounted value of its economic profit is negative. Denote the cutoff productivity for a firm to survive in sector \( k \) by \( \varphi_k \). We shall call this the productivity cutoff for survival or the operating productivity cutoff. Then, the aggregate (exact) price index in sector \( k \) can be rewritten as

\[ P_k = \left\{ \frac{1}{\varphi_k} \int_{\varphi_k}^{\infty} \left[ p_k(\varphi) \right]^{1-\sigma} \frac{g(\varphi)}{1 - G(\varphi)} d\varphi \right\}^{\frac{1}{\sigma-1}} = \theta_k \frac{1}{\sigma^{\frac{1}{\sigma}}} p_k(\bar{\varphi}_k), \]

where \( p_k(\varphi) \equiv \frac{w}{A_k \varphi}, \) \( G(\varphi) \) is the c.d.f. of the distribution of the firm-specific component of productivity \( \varphi \) in the sector and \( g(\varphi) \) is its p.d.f. The function \( G(\varphi) \) is the same for all sectors. Moreover,

\[ p_k(\bar{\varphi}_k) = \frac{w}{\rho A_k \bar{\varphi}_k} \]

where \( \bar{\varphi}_k \) can be interpreted as the “average” productivity in sector \( k \). It can be easily shown that

\[ \bar{\varphi}_k = \left[ \int_{\varphi_k}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1 - G(\varphi)} d\varphi \right]^{\frac{1}{\sigma-1}} = \left( \frac{\gamma}{\gamma - \sigma + 1} \right)^{\frac{1}{\sigma-1}} \bar{\varphi}_k \]  

(5)

where \( G(\varphi_k) \) is the c.d.f. of \( \varphi_k \), which follows a Pareto distribution \( P(1, \gamma) \), as explained above.

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2The assumption \( \gamma > \sigma - 1 \) ensures that, in equilibrium, the size distribution of firms has a finite mean.
The qualitative nature of most of our results will not be affected by this assumption. The zero cutoff profit (ZCP) condition determines the productivity $\bar{\pi}_k$ of the marginal firm that makes zero expected economic profits:

$$\sigma fw = r_k(\bar{\pi}_k) = \frac{b_k w L}{\theta_k} \left( \frac{\bar{\pi}_k}{\bar{\pi}_k} \right)^{\sigma - 1} = \frac{b_k w L}{\theta_k} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) \quad \text{(ZCP)} \quad (6)$$

As more firms enter, the cutoff productivity increases. This in turn lowers the probability of surviving after entry. So, when the cutoff productivity becomes sufficiently high, there will be no more entry. More precisely, the free entry (FE) condition, which relates the cutoff productivity to the entry cost $f_e$, is given by

$$f_e w = p_{in} \bar{v} = [1 - G(\bar{\pi}_k)] \bar{\pi}_k$$

where $p_{in} = 1 - G(\bar{\pi}_k)$ is the ex-ante probability of successful entry; $\bar{\pi}_k \equiv \pi_k(\bar{\pi}_k)$ is the net average profit of a surviving firm, which is equal to $fw \left[ \left( \frac{\bar{\pi}_k}{\bar{\pi}_k} \right)^{\sigma - 1} - 1 \right] = fw \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right)$ according to the ZCP condition (6) and equation (5).

Solving for the above system of 2 equations for 2 unknowns, we can get

$$(\bar{\pi}_k)^\gamma = \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) \frac{f}{f_e} \equiv D_1; \quad \theta_k = \left( \frac{\gamma - \sigma + 1}{\gamma} \right) b_k \frac{L}{\sigma f} \equiv D_2(k) \cdot L$$

Therefore, the fraction of firms that can successfully enter, $1 - G(\bar{\pi}_k)$, is the same across sectors. However, the actual cutoff productivity, $A_k \bar{\pi}_k$, still differs across sectors. From now on, we assume $\left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) \frac{f}{f_e} \geq 1$, in order to avoid corner solution.

**Lemma 1** *In the closed economy, the fraction of firms that can successfully enter is independent of $A_k$. The number of firms in each sector is also independent of $A_k.*

Note that Lemma 1 holds even if $\sigma$ and $\gamma$ differ across sectors.

The intuition for $\bar{\pi}_k$ to be independent of $A_k$ is that an increase in $A_k$ causes a firm’s optimal price to decrease, and as a result, the aggregate price for this sector decreases as well. Consequently, each firm’s optimal price relative to the sectoral aggregate price is unchanged so that the expected profit of each firm does not change. As a result, the fraction of firms that can successfully enter is independent of $A_k$. Note that though the increase in $A_k$ does not affect the number of firms in the sector, it improves consumers’ welfare due to the increased output of each firm.

\(^3\bar{\pi}_k \equiv \pi_k(\bar{\pi}_k) = \frac{r_k(\hat{\pi}_k)}{\sigma} - fw = \frac{1}{\sigma} \left( \frac{\hat{\pi}_k}{\bar{\pi}_k} \right)^{\sigma - 1} r_k(\bar{\pi}_k) - fw = fw \left[ \left( \frac{\hat{\pi}_k}{\bar{\pi}_k} \right)^{\sigma - 1} - 1 \right] = fw \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right). The third equality arises from the fact that $\left( \frac{\hat{\pi}_k}{\bar{\pi}_k} \right)^{\sigma - 1} = \frac{r_k(\hat{\pi}_k)}{\sigma \pi_k(\bar{\pi}_k)}$. The fourth equality stems from the fact that $\sigma fw = r_k(\bar{\pi}_k)$, which is the ZCP condition (6). The fifth equality comes from (5).
3 An Open-economy Model

In this section, we consider a global economy with two countries: Home and Foreign. We attach an asterisk to all the variables pertaining to Foreign. We index sectors such that as the index increases Home’s comparative advantage strengthens. In other words, the sector-specific relative productivity $a(k) \equiv a_k \equiv \frac{A_k}{A_h}$ increases in $k \in [0, 1]$. Therefore, $a'(k) > 0$.

On the demand side, we assume that consumers in both countries have identical tastes:

$$\ln U = \alpha \ln C_h + \int_0^1 b_k \ln C_k dk \quad \text{with} \quad \int_0^1 b_k dk = 1 - \alpha$$

$$C_k = \left( \int_0^\theta c_k(i)^\rho di + \int_0^\theta c_k^*(j)^\rho dj \right)^{\frac{1}{\rho}}$$

On the production side, the labor productivity in the homogeneous good sector are respectively $A_h$ and $A_h^*$ in Home and Foreign. In the rest of the paper, we assume that the homogeneous good sector is sufficiently large so that the homogeneous good is produced in both countries.\(^4\) We also assume that there is no trade cost associated with the homogeneous good. Therefore free trade of homogeneous goods implies that the wage ratio is determined by relative labor productivity in the sector, i.e. $\omega \equiv \frac{w}{w^*} = \frac{A_h^*}{A_h}$, where $w^*$ denotes Foreign’s wage. Without loss of generality, we assume that $\frac{A_h^*}{A_h} = 1$ and normalize by setting $w^* = 1$. Therefore, in equilibrium $w = w^* = 1$. The specification on technology in the differentiated-good sectors is the same as in autarky. The assumptions of a freely traded outside good that is produced by all countries, and Pareto distribution of firm productivity in each differentiated-good sector, greatly simplify the analysis.\(^5\)

The subscript “$dk$” pertains to a domestic firm serving the domestic market in sector $k$, the subscript “$xk$” pertains to a domestic firm serving the foreign market in sector $k$, and the subscript “$k$” pertains to sector $k$ regardless of who serves the market. For the differentiated-good sectors, each firm’s profit-maximizing price in the domestic market is given, as before, by $p_{dk}(j) = \frac{1}{\rho A_k^\phi_k(j)}$. But Home’s exporting firms will set higher prices in the Foreign market due to the existence of an iceberg trade cost, such that $\tau (\tau > 1)$ units of goods have to be shipped from the source in order for one unit to arrive at the destination. Therefore, the optimal export price of a Home-produced good sold in Foreign is given by $p_{xk}(j) = \frac{1}{\rho A_k^\phi_k(j)}$. Similarly, Foreign’s firms’ pricing rules are given by $p_{dk}^*(j) = \frac{1}{\rho A_k^\phi_k(j)}$ and $p_{xk}^*(j) = \frac{\tau}{\rho A_k^\phi_k(j)}$. Here, we assume identical iceberg trade cost $\tau$ for both countries for simplicity. In addition to the iceberg trade cost, the exporting firm has to bear a fixed cost of exporting, $f_x$, which is the same for all firms.

\(^4\)The sufficient condition is $\alpha > \max \left\{ \frac{L_k}{L+L_k}, \frac{L_k}{L+L_k} \right\}$. However, this is just a sufficient, not necessary, condition. In general, we do not need such a strong assumption on $\alpha$, as each country usually both imports and exports differentiated goods. If trade in differentiated goods is close to balanced, $\alpha$ can be much smaller.

\(^5\)In adopting these assumptions, we follow Chaney (2008), who was probably the first to make these assumptions to simplify the analysis.
3.1 Firm entry and exit

According to the firms’ pricing rules, the gross revenue flow and net profit flow of firm $j$ in differentiated sector $k$ from domestic sales for Home’s firms are, respectively:

$$r_{dk}(j) = b_k L \left[ \frac{p_{dk}(j)}{P_k} \right]^{1-\sigma},$$

$$\pi_{dk}(j) = \frac{r_{dk}(j)}{\sigma} - f.$$

The expressions for the corresponding variables for Foreign’s firms, $r_{xk}^*(j)$ and $\pi_{xk}^*(j)$, are defined analogously. The variables $P_k$ and $P_k^*$ are the aggregate price index in sector $k$ of goods sold in Home and Foreign, respectively. Their expressions are given in equation (8) below. Following the same logic, the gross exporting revenue and net profit of firm $j$ in sector $k$ for Home’s firms are, respectively:

$$r_{xk}(j) = b_k L \left[ \frac{p_{xk}(j)}{P_k^*} \right]^{1-\sigma},$$

$$\pi_{xk}(j) = \frac{r_{xk}(j)}{\sigma} - f_x.$$

The expressions for the corresponding variables for Foreign’s firms, $r_{xk}^*(j)$ and $\pi_{xk}^*(j)$, are defined analogously. Let $\varphi_{dk}$ and $\varphi_{xk}$ denote the cutoffs of the firm-specific productivity for domestic sales and exporting respectively of sector $k$ for Home firms; $\varphi_{dk}^*$ and $\varphi_{xk}^*$ denote the corresponding variables for Foreign. Consequently, the mass of exporting firms from Home is equal to:

$$\theta_{xk} = \frac{1 - G(\varphi_{xk})}{1 - G(\varphi_{dk})} \theta_{dk} = \left( \frac{\varphi_{dk}}{\varphi_{xk}} \right)^{\gamma} \theta_{dk}$$

where $\theta_{dk}$ denotes the mass of operating firms in Home. The corresponding expression relating the variables $\theta_{xk}^*$ and $\theta_{dk}^*$ for Foreign are defined analogously. Then, in differentiated-good sector $k$, the mass of varieties available to consumers in Home is equal to

$$\theta_k = \theta_{dk} + \theta_{xk}^*$$

and $\theta_k^*$ is defined analogously. The aggregate price indexes are given by:

$$P_k = (\theta_k)^{\frac{1}{1-\sigma}} p_{dk}(\varphi_k), \quad P_k^* = (\theta_k^*)^{\frac{1}{1-\sigma}} p_{dk}(\varphi_k^*)$$

(8)

where $\varphi_k$ and $\varphi_k^*$ denote the aggregate productivity in differentiated sector $k$ for goods sold in Home and Foreign, respectively. They are given respectively by:

$$\left( \varphi_k \right)^{\sigma-1} = \frac{1}{\theta_k} \left[ \theta_{dk}(\varphi_{dk})^{\sigma-1} + \theta_{xk}^* \left( \tau^{-1} a_k^{\gamma} \varphi_{xk}^{\sigma-1} \right)^{\sigma-1} \right],$$

(9)

$$\left( \varphi_k^* \right)^{\sigma-1} = \frac{1}{\theta_k^*} \left[ \theta_{dk}^*(\varphi_{dk})^{\sigma-1} + \theta_{xk} \left( \tau^{-1} a_k^{\gamma} \varphi_{xk}^{\sigma-1} \right)^{\sigma-1} \right].$$

(10)
where $\tilde{\varphi}_{dk}$ ( $\tilde{\varphi}_{dk}^*$ ) and $\tilde{\varphi}_{xk}$ ( $\tilde{\varphi}_{xk}^*$ ) denote respectively the aggregate productivity level of all of Home’s (Foreign’s) operating firms and Home’s (Foreign’s) exporting firms. The relationships between $\tilde{\varphi}_{dk}$ and $\tilde{\varphi}_{dk}^*$, between $\tilde{\varphi}_{xk}$ and $\tilde{\varphi}_{xk}^*$, between $\tilde{\varphi}_{xk}$ and $\tilde{\varphi}_{xk}^*$, and between $\tilde{\varphi}_{xk}$ and $\tilde{\varphi}_{xk}^*$, are exactly the same as in the closed economy, namely equation (5). That is, $\tilde{\varphi}_{sk} = \left( \frac{\gamma}{\gamma - \sigma + 1} \right) \frac{1}{\sigma - 1} \tilde{\varphi}_{sk}$ and $\tilde{\varphi}_{sk}^* = \left( \frac{\gamma}{\gamma - \sigma + 1} \right) \frac{1}{\sigma - 1} \tilde{\varphi}_{sk}^*$ for $s = x, d$. From the above equations, it is obvious that these aggregate productivity measures as well as aggregate price indexes are functions of ($\tilde{\varphi}_{dk}$, $\tilde{\varphi}_{dk}^*$, $\tilde{\varphi}_{xk}$, $\tilde{\varphi}_{xk}^*$, $\theta_{dk}$, $\theta_{dk}^*$). As will be shown below, as long as $\frac{b}{d}$ is sufficiently large, an entering firm will produce only if it can generate positive present-discounted profit by selling domestically, and export only if it can generate positive present-discounted profit by selling abroad. Then we have the following four zero cutoff profit conditions

$$r_{dk} (\tilde{\varphi}_{dk}) = b_k L \left( P_k \rho A_k \tilde{\varphi}_{dk} \right)^{\sigma - 1} = \sigma f$$  \hspace{1cm} (11)

$$r_{dk}^* (\tilde{\varphi}_{dk}^*) = b_k L \left( \frac{P_k}{\sigma} \rho A_k \tilde{\varphi}_{dk}^* \right)^{\sigma - 1} = \sigma f$$  \hspace{1cm} (12)

$$r_{xk} (\tilde{\varphi}_{xk}) = b_k L \left( \frac{P_k}{\sigma} \rho A_k \tilde{\varphi}_{xk} \right)^{\sigma - 1} = \sigma f_x$$  \hspace{1cm} (13)

$$r_{xk}^* (\tilde{\varphi}_{xk}^*) = b_k L \left( \frac{P_k}{\sigma} \rho A_k \tilde{\varphi}_{xk}^* \right)^{\sigma - 1} = \sigma f_x$$  \hspace{1cm} (14)

Define $\tilde{\pi}_k$ and $\tilde{\pi}_k^*$ as the average profit flow of a surviving firm in sector $k$ in Home and Foreign respectively. It can be easily shown that:

$$\tilde{\pi}_k = \pi_{dk} (\tilde{\varphi}_{dk}) + \left[ 1 - G (\tilde{\varphi}_{xk}) \right] \pi_{xk} (\tilde{\varphi}_{xk}) = \frac{\sigma - 1}{\gamma - \sigma + 1} \left[ f + \left( \frac{\tilde{\varphi}_{dk}}{\tilde{\varphi}_{xk}} \right)^\gamma f_x \right]$$

$$\tilde{\pi}_k^* = \pi_{dk}^* (\tilde{\varphi}_{dk}^*) + \left[ 1 - G (\tilde{\varphi}_{xk}^*) \right] \pi_{xk}^* (\tilde{\varphi}_{xk}^*) = \frac{\sigma - 1}{\gamma - \sigma + 1} \left[ f + \left( \frac{\tilde{\varphi}_{dk}^*}{\tilde{\varphi}_{xk}^*} \right)^\gamma f_x \right].$$

These are analogous to the equation shown in footnote 3 for the closed economy. The potential entrant will enter if her expected post-entry present-discounted profit is above the cost of entry. Hence, the Free Entry (FE) conditions for Home and Foreign are, respectively

$$f_e = \left[ 1 - G (\tilde{\varphi}_{dk}) \right] \tilde{\pi}_k = \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) \left[ f \cdot \left( \tilde{\varphi}_{dk} \right)^{-\gamma} + f_x \cdot \left( \tilde{\varphi}_{xk} \right)^{-\gamma} \right]$$  \hspace{1cm} (15)

$$f_e = \left[ 1 - G (\tilde{\varphi}_{dk}^*) \right] \tilde{\pi}_k^* = \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) \left[ f \cdot \left( \tilde{\varphi}_{dk}^* \right)^{-\gamma} + f_x \cdot \left( \tilde{\varphi}_{xk}^* \right)^{-\gamma} \right]$$  \hspace{1cm} (16)

---

6 The derivation of the above two equations are available from the corresponding author’s homepage at http://ihome.ust.hk/~elai/ or upon request.

7 The condition is $\frac{b}{d} > \max \{ \frac{\beta_x}{\beta}, \frac{L_x}{L} \}$. If this condition is not satisfied, then there exist some sectors in which all firms export (besides serving the domestic market).

8 $\tilde{\varphi}_{dk} \equiv \pi_{dk} (\tilde{\varphi}_{dk}) = \frac{r_{dk} (\tilde{\varphi}_{dk})}{\sigma - 1} - f = \left( \frac{\tilde{\varphi}_{dk}}{\sigma - 1} \right)^{-1} - f = \left( \frac{\tilde{\varphi}_{dk}}{\sigma - 1} \right)^{-1} - f = \left( \frac{\tilde{\varphi}_{dk}}{\sigma - 1} \right)^{-1} - f \cdot \left( \frac{\tilde{\varphi}_{dk}}{\sigma - 1} \right)^{-1} = f \cdot \frac{\sigma - 1}{\gamma - \sigma + 1}$. The third equality arises from the fact that $\left( \frac{\tilde{\varphi}_{dk}}{\sigma - 1} \right)^{-1} = \frac{\tilde{\varphi}_{dk} (\tilde{\varphi}_{dk})}{\sigma - 1}$. The fourth equality comes from the fact that $\sigma f = r_{dk} (\tilde{\varphi}_{dk})$, which is the ZCP condition above. The fifth equality comes from equation (5). Furthermore, $\tilde{\varphi}_{sk} = f_x \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right)$ can be derived from similar steps as above by replacing the subscript “$d$” by “$x$” and the variable $f$ by $f_x$. Finally, $1 - G (\varphi) = \varphi^{-\gamma}$. 

9
3.2 General equilibrium

Assuming that both countries produce in sector \( k \), given the wage ratio \( A_h/A_h^* = 1 \), we can solve for \( (\zeta_{dk}, \zeta_{dk}^*, \zeta_{xk}, \zeta_{xk}^*, \theta_{dk}, \theta_{dk}^*) \) from the four zero cutoff profit conditions and two free entry conditions (11) to (16) since the aggregate prices are functions of these six variables (for details, please refer to Appendix A). The solutions are given below.

\[
(\zeta_{dk}) = D_1 \left[ \frac{B - B^{-1}}{B - (a_k)^{\gamma}} \right] \]

\[
(\zeta_{dk}^*) = D_1 \left[ \frac{B - B^{-1}}{B - (a_k)^{\gamma}} \right] \]

\[
\zeta_{xk} = \left( \frac{B f_x}{f} \right)^{\frac{1}{\gamma}} \zeta_{dk} \]

\[
\zeta_{xk}^* = \left( \frac{B f_x}{f} \right)^{\gamma} a_k \zeta_{dk} \]

\[
\theta_{dk} = D_2(k) \left[ \frac{BL - B(a_k)^{\gamma} - L^*}{B - B^{-1}} \right] \]

\[
\theta_{dk}^* = D_2(k) \left[ \frac{BL^* - B(a_k)^{\gamma} - L}{B - B^{-1}} \right] \]

where \( B \equiv \tau^\gamma \left( \frac{L^*}{f} \right)^{\frac{1}{\gamma}-1} \). The variable \( B \) can be interpreted as summary measures of trade barriers; \( a_k \) can be interpreted as competitiveness of Home in differentiated goods sector \( k \). Recall that \( a_k^*(k) > 0 \) is assumed.

In this paper, we impose the condition \( \frac{L^*}{f} > \max \{ \frac{L^*}{f}, \frac{L^*}{f} \} \) so as to ensure that some firms produce exclusively for their domestic market in all sectors.\(^9\), \(^10\)

According to equations (21) and (22) Home’s firms will exit sector \( k \) when \( \theta_{dk} \leq 0 \), and Foreign’s firms will exit the sector if \( \theta_{dk}^* \leq 0 \). This implies that \( B \frac{B - B(a_k)^{\gamma}}{B(a_k)^{\gamma} - 1} < \frac{L^*}{f} < B \frac{B - B(a_k)^{\gamma}}{B(a_k)^{\gamma} - 1} \) is needed for both countries to produce positive outputs in sector \( k \), otherwise there will be complete dominance by one country in the sector and one-way trade. Rearranging these inequalities, we can sort the sectors into three types according to Home’s strength of comparative advantage. Home will not produce in

\(^9\)The proof is straightforward. From Table 1, we see that \( \zeta_{dk} < \zeta_{xk} \iff f > \frac{1}{B(a_k)^{\gamma}} \). Similarly, \( \zeta_{dk} < \zeta_{xk} \iff f > \frac{1}{B(a_k)^{\gamma}} \). Equations (21) and (22) imply that \( \frac{1}{B(a_k)^{\gamma}} \leq \frac{L^*}{f} \) and \( \frac{1}{B(a_k)^{\gamma}} \leq \frac{L^*}{f} \) for \( k \in [k_1, k_2] \), where \( \theta_{dk} \geq 0 \) and \( \theta_{dk}^* \geq 0 \). Hence \( \frac{L^*}{f} > \max \{ \frac{L^*}{f}, \frac{L^*}{f} \} \) is a sufficient condition for \( \zeta_{dk} < \zeta_{xk} \) and \( \zeta_{dk} < \zeta_{xk}^* \) for all two-way trade sectors.

In addition, Table 1 shows that \( \frac{L^*}{f} > \max \{ \frac{L^*}{f}, \frac{L^*}{f} \} \) is also a sufficient condition for \( \zeta_{dk} < \zeta_{xk} \) and \( \zeta_{dk} < \zeta_{xk}^* \) (whenever the country produces) for all one-way trade sectors.

\(^10\)In a one-sector model, Melitz (2003) imposes the condition \( \tau^{n-1} f_x > f \) so as to ensure that some firms produce exclusively for their domestic market in both countries. In this paper, we adopt a more stringent standard so as to ensure that some firms sell exclusively to their domestic market in all sectors. It’s obvious that \( B > 1 \) under our condition.
sector \( k \) iff \( k \leq k_1 \), where \( k_1 \) satisfies

\[
(a_{k_1})^\gamma = \frac{B \left( \frac{L}{L^*} + 1 \right)}{B^2 + 1};
\]  
(23)

and Foreign will not produce in sector \( k \) iff \( k \geq k_2 \), where \( k_2 \) satisfies

\[
(a_{k_2})^\gamma = \frac{B^2 L^* + 1}{B \left( \frac{L}{L^*} + 1 \right)}.
\]  
(24)

Therefore, the solutions to (17)-(22) are valid if and only if \( k_2 \) \((k_1, k_2)\) implies that \((a_k)^\gamma \in (\frac{1}{B}, B)\) for any possible GDP ratio \( L/L^* \), which ensures that the productivity cutoffs will never reach the corner for the sectors in which both countries produce.

When \( k \notin (k_1, k_2) \), one of the countries dominates sector \( k \). In that case, there is no interior solution to some of the equations in the system above, as no firm in the other country has incentive to enter the market, which means that the number of firms in that country solved from the system is negative. Therefore, a different set of equations need to be solved for this case. Without loss of generality, we consider the **Home-dominated sectors**. As there is no competition from Foreign’s firms when Home’s firms sell in Foreign, the aggregate price indexes become

\[
P_k = (\theta_{dk})^{\frac{1}{\gamma}} \frac{1}{\rho A_k \varphi_{dk}}
\]

\[
P_k^* = (\theta_{xk})^{\frac{1}{\gamma}} \frac{\tau}{\rho A_k \varphi_{xk}}
\]

Accordingly, the two zero cutoff conditions for Home (11) and (13) continue to hold.

As the Free entry condition (15) for Home firms continues to hold, solving the diminished system of three equations (11), (13), (15) for three unknowns, we have

\[
\theta_{dk} = \frac{b_k L}{\sigma f} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) = D_2 (k) L
\]

\[
\theta_{xk} = \frac{b_k L^*}{\sigma f_x} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) = D_2 (k) \frac{f}{f_x} L^*
\]

\[
(\varphi_{dk})^\gamma = \frac{L + L^*}{L} D_1.
\]

Furthermore, we can easily obtain \((\varphi_{xk})^\gamma = (L + L^*) \frac{f_{x}}{f} D_1\) by noting that \(\theta_{xk} = \frac{1 - G(\varphi_{xk})}{1 - G(\varphi_{dk})} \theta_{dk}\). An analogous set of solutions for the Foreign-dominated sectors can be obtained.\(^\text{12}\) \(^\text{13}\)

\(^\text{11}\) Because \(B^2 + 1 > B \left( \frac{L}{L^*} + 1 \right) \) holds as long as \(B > 1\), we always have \(k_1 < k_2\).

\(^\text{12}\) They are: \(\theta_{dk} = \frac{b_k L}{\sigma f} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) = D_2 (k) L^*\); \(\theta^*_{xk} = \frac{b_k L}{\sigma f_x} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) = D_2 (k) \frac{f}{f_x} L^*\); and \((\varphi^*_{dk})^\gamma = \frac{L + L^*}{L} D_1\).

\(^\text{13}\) The uniqueness of the above equilibrium is proved in an appendix posted on the corresponding author’s homepage at http://ihome.ust.hk/~elai/ or upon request.
Proposition 1 In sectors $k \in [k_2, 1]$, where Home has the strongest comparative advantage, only Home produces, and there is one-way trade. An analogous situation applies to Foreign in sectors $k \in [0, k_1]$. In sectors $k \in (k_1, k_2)$, where neither country has strong comparative advantage, both countries produce, and there is two-way trade.

We show the three zones of international specialization in Figure 1. The upward sloping curve (including the dotted portions) corresponds to equation (21), while the downward sloping curve (including the dotted portions) corresponds to equation (22). The horizontal portion of $\theta_{dk}$ in the diagram corresponds to the equation for $\theta_{dk}$ above when Home dominates sector $k$ completely. The horizontal portion of $\theta^*_{dk}$ corresponds to the analogous equation for Foreign.

Figure 1. Three Zones of International Specialization (assumptions: (i) expenditure shares are equal across sectors; (ii) $L < L^*$).

4 Opening up to Trade

In this section, we analyze how opening trade between the two countries impacts the economy of each country, e.g., the productivity cutoffs, the mass of producing and exporting firms, as well as welfare. Before proceeding with the analysis, it is helpful to list the solutions to the relevant variables corresponding to the three types of sectors in Table 1.
Table 1: Solution of the System

\[ D_1 = \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) \frac{f}{f_c} \quad D_2 (k) = \left( \frac{\gamma - \sigma + 1}{\gamma} \right) \frac{b_k}{\sigma f} \quad (a_{k_1})^\gamma = \frac{B (\frac{L^*}{L} + 1)}{B^2 (\frac{L^*}{L} + 1)} \quad (a_{k_2})^\gamma = \frac{B^2 (\frac{L^*}{L} + 1)}{B (\frac{L^*}{L} + 1)} \]

4.1 Impacts on productivity cutoffs

In this subsection, we analyze how trade affects the productivity cutoffs from two aspects: within sector and across sectors. First, we look at how trade integration changes the cutoffs within a certain sector. As a result, we find that the impacts of trade integration on productivity cutoffs are the same as in Melitz (2003) in all sectors. Then we compare the cutoffs across sectors upon trade integration. We add a subscript c to all the parameters pertaining to autarky (c=closed economy). It has been shown in Section 2 that the autarky productivity cutoff for survival in Home and Foreign is given by \((\varphi_{dk})^\gamma = (\varphi_{ek})^\gamma = \left( \frac{\sigma - 1}{\gamma - \sigma + 1} \right) \frac{f}{f_c} = D_1\). If both countries produce, then the equilibrium cutoffs for survival are given by (17) and (18). As \((a_k)^\gamma \in \left( \frac{1}{B}, B \right)\), we have \(\varphi_{dk} > \varphi_{ek}\) and \(\varphi_{dk} > \varphi_{ck}\).

Recall that if only one country produces, the equilibrium operating cutoffs are given by:

\[ (\varphi_{dk})^\gamma = \frac{L + L^*}{L} - D_1 > (\varphi_{ek})^\gamma \quad \text{if only Home produces} \]

\[ (\varphi_{dk})^\gamma = \frac{L + L^*}{L^*} - D_1 > (\varphi_{ck})^\gamma \quad \text{if only Foreign produces} \]

Hence, the least productive firms in all sectors will exit the market after trade integration. As a result, resources will be reallocated to the most productive firms. Furthermore, \(\varphi_{dk} > \varphi_{ek}\), implies that \(\varphi_{dk} > \varphi_{ck}\), and \(\varphi_{dk} > \varphi_{ek}\) implies that \(\varphi_{dk} > \varphi_{ck}\). Therefore, the average productivity in any sector \(k\) is higher under trade integration than in autarky. Thus we generalize Melitz’s result to a setting where there exist endogenous intra-industry trade and inter-industry trade in a single model.
In the closed economy, the cutoffs for survival are identical across sectors. However, this is not true any more in the open economy. In the sectors where both countries produce, the equilibrium cutoff for survival is an increasing function of the sectoral comparative advantage. More precisely, as $a_k$ increases, $\bar{\varphi}_{dk}$ rises but $\bar{\varphi}_{dk}^*$ falls, and, following the free entry conditions (15) and (16), $\bar{\varphi}_{xk}$ falls but $\bar{\varphi}_{xk}^*$ rises. Thus, we have

**Proposition 2** In sectors where both countries produce, for a given country, a sector with stronger comparative advantage has a higher fraction of domestic firms that export and higher fraction of revenue derived from exporting.

Moreover, $\bar{\varphi}_{xk}^* > \bar{\varphi}_{xk} > \bar{\varphi}_{dk} > \bar{\varphi}_{dk}^*$ iff Home is more competitive in sector $k$ ($a_k > 1$), while $\bar{\varphi}_{xk} > \bar{\varphi}_{xk}^* > \bar{\varphi}_{dk} > \bar{\varphi}_{dk}^*$ iff $a_k < 1$. This result and Proposition 2 are summarized by Figure 2 below.

![Graph showing how productivity cutoffs vary across sectors](image)

**Figure 2. How productivity cutoffs vary across sectors**

### 4.2 Impacts on the masses of firms

In this subsection, we focus on how trade will affect the mass of firms in each sector. As in the previous subsection, a subscript $c$ denotes all the variables referring to the closed economy.

Recall that if both countries produce, then

$$
\theta_{dk} = D_2 (k) \frac{BL - B^{-(a_k)} L^*}{B - B^{-1}} < D_2 (k) L = \theta_{ck}; \quad \theta_{dk}^* = D_2 (k) \frac{BL^* - B^{-(a_k)} L}{B - B^{-1}} < D_2 (k) L^* = \theta_{ck}^*.
$$
If only one country produces, then
\[ \theta_{dk} = \frac{b_k L}{\sigma f} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) = D_2(k) L = \theta_{ck} \quad \text{if only Home produces} \]
\[ \theta^*_{dk} = \frac{b_k L^*}{\sigma f} \left( \frac{\gamma - \sigma + 1}{\gamma} \right) = D_2(k) L^* = \theta^*_{ck} \quad \text{if only Foreign produces} \]

We summarize the above findings in the following proposition:

**Lemma 2** In sectors where only Home produces under trade, the number of domestic firms in Home is the same as in autarky. In sectors where both countries produce under trade, the number of domestic firms in each sector decreases in Home after opening up to trade.

Obviously, this proposition applies equally to Foreign.

### 4.3 Impacts on welfare

For sectors in which **both countries produce**, i.e. when \((a_k)^\gamma \in \left(0, \frac{B(L^*+1)}{B^2 L} \right)\), we can write Home’s aggregate price index in the sector as:

\[ P_k = (\theta_{dk} + \theta^*_{xk})^{\frac{1}{1-\sigma}} p_{dk}(\phi_{dk}) = \left( \theta_{dk} + \theta^*_{xk} \frac{f_x}{f} \right)^{\frac{1}{1-\sigma}} p_{dk}(\phi_{dk}) \]

Substituting the equilibrium values of \(\theta_{dk}, \theta^*_{xk}, \theta^*_{ck}, \theta_{xk}\) into the above equation, we find that \(\theta_{dk} + \theta^*_{xk} \frac{f_x}{f} = \theta_{ck}\). Therefore, we can simplify the price index as:

\[ P_k = (\theta_{ck})^{\frac{1}{1-\sigma}} \frac{1}{\rho A_k \phi_{dk}} \]

Then, Home’s real wage in terms of the aggregate good in this sector (hereinafter we shall refer to real wage in terms of the aggregate good in sector k as the “real wage in terms of good k”) is given by:

\[ \frac{1}{P_k} = (\theta_{ck})^{\frac{1}{1-\sigma}} \rho A_k \phi_{dk} = \left[ \frac{B - B^{-1}}{B - (a_k)^\gamma} \right]^{\frac{1}{\sigma}} \frac{1}{P_{ck}} > \frac{1}{P_{ck}} \quad (25) \]

In a sector where **Foreign completely dominates**, i.e. when \((a_k)^\gamma \in \left(0, \frac{B(L^*+1)}{B^2 L} \right)\), Home’s real wage in terms of the aggregate good in this sector is given by:

\[ \frac{1}{P_k} = (\theta^*_{xk})^{\frac{1}{1-\sigma}} \rho A_k \phi_{xk} \frac{1}{\rho A_k \phi_{xk}} = a_k^{-1} B^{-\frac{1}{\tau}} \left( \frac{L + L^*}{L} \right)^{\frac{1}{\tau}} \frac{1}{P_{ck}} > \frac{1}{P_{ck}} \quad (26) \]

In a sector where **Home completely dominates**, i.e. when \((a_k)^\gamma \in \left(\frac{B^2 L^*}{(L^*+1)}, \infty \right)\), Home’s real wage in terms of the aggregate good in this sector is given by:

\[ \frac{1}{P_k} = (\theta_{dk})^{\frac{1}{\tau-1}} \rho A_k \phi_{dk} = \left( \frac{L + L^*}{L} \right)^{\frac{1}{\tau}} \frac{1}{P_{ck}} > \frac{1}{P_{ck}} \quad (27) \]
Therefore, the welfare increases after trade integration. The following proposition and Figure 3 summarize the analysis above.

**Proposition 3** Welfare increase in both countries after they open up to trade with each other.

![Diagram](image)

**Figure 3.** Welfare Impact of Trade Integration \( w = w^* = 1 \) by assumption and normalization

In the next section, we perform comparative statics concerning the effects of trade liberalization. Unlike Dornbusch et al (1977), the relative wage is directly determined by the relative productivity in the homogeneous-good sector in our model.\(^{14}\)

## 5 Trade liberalization

### 5.1 Theoretical predictions

Trade liberalization is interpreted as a reduction of the iceberg trade cost \( \tau \), which lowers \( B = \tau^\gamma \left( \frac{L}{L^*} \right)^{\frac{\gamma-\sigma+1}{\gamma-1}} \). Without loss of generality, we only focus on the case when \( L/L^* \geq 1 \). With a slight abuse of language, “the real wage in the sector” shall mean “the real wage in terms of the aggregate good in the sector”.

As (25) and (17) show, in the sectors where both countries produce, the real wages in Home and Foreign in terms of good \( k \), \( 1/P_k \) and \( 1/P_k^* \), just depend on the production cutoff \( \tau_{dk} \) and \( \tau_{dk}^* \) respectively. Thus, \( \frac{d(\tau_{dk})^\gamma}{dB} > 0 \Leftrightarrow \frac{d(1/P_k)}{dB} > 0 \) and \( \frac{d(\tau_{dk})^\gamma}{dB} > 0 \Leftrightarrow \frac{d(1/P_k^*)}{dB} > 0 \). We calculate

\(^{14}\)We have also tried the version without the homogeneous sector, and relative wage is determined by balance of trade, as in Dornbusch et al. (1977). In that case, the effect on welfare is highly ambiguous, but still the results are very different from Dornbusch et al. (1977), unless we made the fully symmetric assumption like Okubo (2009).
\[
\frac{d (\varphi_{dk})^\gamma}{dB} = \frac{2B^{-1} - (1 + B^{-2}) (a_k)^\gamma}{[B - (a_k)^\gamma]^2}
\]  \tag{28}
\[
\frac{d (\varphi_{dxk})^\gamma}{dB} = \frac{2B^{-1} - (1 + B^{-2}) (a_k)^{-\gamma}}{[B - (a_k)^{-\gamma}]^2}
\]  \tag{29}

which shows that \(\varphi_{dk}\) increases with \(B\) (and \(\varphi_{xk}\) decreases with \(B\) according to (15)) if and only if \((a_k)^\gamma < \frac{2B}{1+B^2}\). Moreover, \(\varphi_{dk}^\gamma\) increases with \(B\) (and \(\varphi_{xk}^\gamma\) decreases with \(B\) according to (16)) if and only if \((a_k)^\gamma > \frac{1+B^2}{2B}\). Recalling that \((a_{k1})^\gamma = \frac{B(L/L^*+1)}{B^2(L/L^*+1)}\) and \((a_{k2})^\gamma = \frac{B^2(L/L^*+1)}{B(L/L^*+1)}\), and comparing them with the above thresholds, we can obtain the following conclusions. (i) When \(L = L^*\), \(k_1\) and \(k_2\) exactly coincide with these thresholds, meaning that in the two-way trade sectors, \(\varphi_{dk}\) (as well as real wage in sector \(k\)) always decreases with \(B\), and \(\varphi_{xk}\) always increases with \(B\). In other word, Melitz’s predictions hold in this case. (ii) As \(L/L^*\) increases above one, \(k_1\) decreases, and there exist some two-way trade sectors \(k \in \left[k_1, \frac{2B}{1+B^2}\right]\), in which \(\varphi_{dk}\) increases with \(B\) and \(\varphi_{xk}\) decreases with \(B\). That is, trade liberalization leads to a decrease in the productivity cutoff for survival but an increase in the exporting cutoff. Moreover, the real wage in these sectors also fall. These are all opposite to the predictions of the original Melitz model. Thus, we call this the “counter-Melitz effect”. (iii) As \(L/L^*\) increases above one, \(k_2\) decreases, and so there does not exist any sector in which \(\varphi_{dk}^\gamma\) increases with \(B\) or \(\varphi_{xk}^\gamma\) decreases with \(B\). Thus, the counter-Melitz effect does not exist in any sector in the smaller country.

For a more detailed mathematical analysis of the impacts of trade liberalization, refer to equations (25) to (27) and to Appendix B.

Figure 4 summarizes the effects graphically. The diagram shows the welfare effects of trade liberalization. The \(k_1\) and \(k_2\) curves show the pattern of international specialization for any given \(L/L^*\). (Recall that \(L/L^* \geq 1\) is assumed.) The zone to the left of the \(k_1\) curve corresponds to sectors completely dominated by Foreign. The zone to the right of the \(k_2\) curve corresponds to the sectors completely dominated by Home. The downward sloping \(k_1\) curve indicates that as the relative size of Home becomes larger, it can profitably produce in more sectors. This shows the home market effect as explained by Krugman (1980) — the firms located in the larger country has the advantage of saving the trade costs of serving the larger market, which more than compensates for its cost disadvantage relative to the firms located in the smaller country in the same sector. On the other hand, the downward sloping \(k_2\) curve shows that Foreign, the smaller country, can profitably produce in fewer sectors as the relative size of Home increases. This reminds us of the result in Markusen and Venables (2000), in which they find that the larger country can export a good in which it has comparative disadvantage because of home market effect. As will be shown later, this explains why the “counter-Melitz effect” can only occur in the larger country.

The figure also shows that, for any given value of \(L/L^*\), the signs of the real-wage effect of trade liberalization on Home and Foreign in different sectors. The upper sign inside a rectangle indicates the sign of Home’s change in real wage in terms of good \(k\) due to an infinitesimal decrease in \(\tau\), and the lower sign indicates the sign of Foreign’s change in real wage in terms of good \(k\). For example, for \(L/L^* < B^2\), when there is an infinitesimal reduction of \(B\) (via reduction in \(\tau\)), Home’s real wage
(i) increases in terms of goods in the Foreign-dominated sectors (the zone to the left of the $k_1$ curve), (ii) decreases in the two-way trade sectors to the right of the $k_1$ curve but to the left of the vertical line $(a_k) = \frac{2B}{1+B^2}$ (this corresponds to the slanted-hatched zone), (iii) increases in the two-way trade sectors to the right of the vertical line $(a_k) = \frac{2B}{1+B^2}$ but to the left of the $k_2$ curve (this corresponds to the vertically-hatched zone), and (iv) does not increase or decrease in the Home-dominated sectors (the zone to the right of the $k_2$ curve).

Note that Figure 4 indicates that there is a “counter-Melitz effect” for Home (the larger country) in the slanted-hatched zone, in the sense that $\varphi_{dk}$ decreases and $\varphi_{zk}$ increases in response to trade liberalization. However, the Melitz effect dominates in Home in the vertically-hatched zone, in the sense that $\varphi_{dk}$ increases and $\varphi_{zk}$ decreases in response to trade liberalization. These results reflect the algebraic derivation above and in Appendix B. Note that there is no counter-Melitz effect for Foreign, the smaller country.
Figure 4. Welfare Effects of Trade Liberalization (infinitesimal reduction of $B$ through a reduction of $\tau$). In each region, the upper sign inside the rectangle indicates the welfare change of Home and the lower sign indicates the welfare change of Foreign. The short horizontal arrows indicate the movement of lines as $B$ falls.

As $B$ decreases, the curves for $k_1$ and $k_2$, as well as the vertical lines corresponding to $(a_k)^\gamma = \frac{2B}{1+B^2}$ and $(a_k)^\gamma = \frac{B^2+1}{2B}$, will all shift, with the directions of shifts shown by the small horizontal arrows in Figure 4. For any given $L/L^*$, as $\tau$ (and therefore $B$) decreases from a large number, $k_1$ increases, $k_2$ first decreases then increases, $(a_k)^\gamma = \frac{2B}{1+B^2}$ increases while $(a_k)^\gamma = \frac{B^2+1}{2B}$ decreases.

Depending on the range of $[a_0, a_1]$ and the value of $L/L^*$, it is possible that the $k_1$ or $k_2$ curve (or both) may situate outside the range $k \in [0, 1]$ for some or all values of $L/L^*$. For example, if $a_{k_1} < a_0$ for a given value of $L/L^*$, then no Foreign-dominated sector exist for that value of $L/L^*$. This is because as $L/L^*$ gets sufficiently large, the home-market effect in Home gets so strong that Home can compete even in the sector in which it has the weakest comparative advantage, namely sector $k = 0$. 
The above discussion and Figure 4 can be summarized by the following lemma and proposition:

**Lemma 3** When the two countries are of the same size, trade liberalization improves the real wages in all two-way trade sectors in both countries.

**Proposition 4** Suppose Home is larger than Foreign. In the sectors where Home has the strongest comparative disadvantage but still produces, there is a counter-Melitz effect in the sense that $\bar{\varphi}_{dk}$ decreases while $\varphi_{xk}$ increases in the face of trade liberalization, leading to a reduction in real wage in these sectors. Foreign, the smaller country, will never suffer from real wage reduction in any sector upon trade liberalization as it will never experience any counter-Melitz effect.

Proposition 4 deserves more discussion, as it highlights one of the most important results of this paper. If Home is the larger country, the sectors in which its real wage decreases upon trade liberalization are defined by $\{ k \mid (a_{k1})^\gamma < (a_k)^\gamma < (a_{k2})^\gamma \text{ and } (a_k)^\gamma < \frac{2B}{1+B^2} \}$. The first condition indicates that the sector is a two-way trade sector. The second condition indicates that the sector is among those in which the larger country has the weakest comparative advantage. The two conditions combined say that, in the sectors where the larger country has the weakest comparative advantage yet still produces, Home’s real wage decreases with trade liberalization. In other words, there is a **counter-Melitz effect** in these sectors, i.e. $\bar{\varphi}_{dk}$ decreases while $\varphi_{xk}$ increases in the face of trade liberalization, leading to a decrease in the average productivity of firms serving the Home market, thus lowering real wage in that sector. We can explain the existence of the counter-Melitz effect by decomposing the total effect of trade liberalization into two effects: the Melitz effect and the inter-sectoral resource allocation (IRA) effect. We shall analyze from the perspective of Home and Home’s firms.

- Note that
  \[
  n_k = \frac{\theta_{dk}}{1 - G(\bar{\varphi}_{dk})} = \theta_{dk} (\bar{\varphi}_{dk})^\gamma = D_1 D_2 (k) \left[ \frac{BL}{B - (a_k)^\gamma} - \frac{L^*}{B(a_k)^\gamma - 1} \right]
  \]  
  and recall
  \[
  \theta_{dk} = D_2 (k) \left[ \frac{BL - \frac{B-(a_k)^\gamma}{B(a_k)^\gamma-1}L^*}{B - B^{-1}} \right] \tag{21}
  \]
  \[
  (\bar{\varphi}_{dk})^\gamma = D_1 \left[ \frac{B - B^{-1}}{B - (a_k)^\gamma} \right] \tag{17}
  \]
  \[
  \frac{d(\bar{\varphi}_{dk})^\gamma}{dB} = \frac{2B^{-1} - (1 + B^{-2}) (a_k)^\gamma}{[B - (a_k)^\gamma]^2} \tag{28}
  \]
  \[
  \frac{1}{P_k} = \left[ \frac{B - B^{-1}}{B - (a_k)^\gamma} \right]^{\frac{1}{\gamma}} \frac{1}{P_{ck}} \tag{25}
  \]
• When $L = L^*$, and $a_k = 1$, $\forall k$, our model collapses to the Melitz model with a homogenous-good sector. (30) $\Rightarrow n_k = D_1 D_2 (k) L$. With trade liberalization, the numbers of potential entrants $n_k$ and $n_k^*$ will remain unchanged. Note that (17) $\Rightarrow (\varphi_{dk})^\gamma = D_1 (B+1)$. Therefore, $\frac{d(\varphi_{dk})^\gamma}{dB} < 0$. That is, real wages in all sectors rise with trade liberalization. As a result, the export revenue of a typical exporting firm will increase as trade cost falls. This creates pressure for both $\varphi_{xk}$ and $\varphi_{zk}^*$ to decrease. Meanwhile, this will force the least productive firms in each country to exit, as there are more firms exporting to the domestic market. This creates pressure for both $\varphi_{dk}$ and $\varphi_{dk}^*$ to increase. The increase in average productivity in all sectors lead to increase in real wage in all sectors. This is the Melitz effect.

• Next, we allow $a_k$ to deviate from 1 for some $k$, but keep $L = L^*$. This creates comparative advantage for Home in some sectors and comparative disadvantage for Home in other sectors. Note that (28) holds only if sector $k$ satisfies the constraint for two-way trade, given by $(a_{k1})^\gamma < (a_k)^\gamma < (a_{k2})^\gamma$. Observe that (i) (23) $\Rightarrow (a_{k1})^\gamma < (a_k)^\gamma$ is equivalent to $2B^{-1} - (1 + B^{-2}) (a_k)^\gamma < 0$; therefore, (28) $\Rightarrow \frac{d(\varphi_{dk})^\gamma}{dB} < 0$ for all two-way trade sectors. In other words, trade liberalization always increases the real wage in a two-way trade sector. That is, the Melitz effect dominates in all two-way trade sectors as long as $L = L^*$. (ii) (28) $\Rightarrow \left| \frac{d(\varphi_{dk})^\gamma}{dB} \right|$ increases with $a_k$. That is, the dominance of the Melitz effect in a sector decreases as the comparative advantage of Home diminishes. (iii) (23) $\Rightarrow k = k_1$, $2B^{-1} - (1 + B^{-2}) (a_k)^\gamma = 0$, implying that $\frac{d(\varphi_{dk})^\gamma}{dB} = 0$. That is, the Melitz effect is completely offset in sector $k_1$, the sector in which Home has the least comparative advantage yet still produces. What happens to make $\left| \frac{d(\varphi_{dk})^\gamma}{dB} \right|$ increase with $a_k$? The effect comes from re-allocation of resources (labor) between sectors as $B$ decreases, which we call inter-sectoral resource allocation (IRA) effect. Trade liberalization leads to resources in Home (as well in Foreign) being re-allocated away from the differentiated-good sectors in which it has comparative disadvantage to ones in which it has comparative advantage (and possibly the homogeneous good sector). Therefore, $n_k$ decreases and $n_k^*$ increases in the sectors in which Home has comparative disadvantage. As $n_k^*$ increases, Foreign’s market becomes more competitive (as there are more firms in Foreign) and so $r_{xk}(\varphi)$ decreases for all $\varphi$. This creates pressure for an increase in $\varphi_{zk}$ (i.e. only the more productive Home firms can profitably export now). As $n_k$ decreases, $\theta_{dk}$ also decreases. This leads to the expansion of the sizes of the surviving Home firms. Thus, $r_{dk}(\varphi)$ increases for all $\varphi$. This creates pressure for a decrease in $\varphi_{dk}$ as some less productive firms which were expected to be unprofitable before can be expected to be profitable now. The variables move in opposite directions in the sectors in which Home has comparative advantage. In other words, the IRA effect reinforces the Melitz effect in the sectors in which Home has comparative advantage but counteracts the Melitz effect in the sectors in which it has comparative disadvantage. Observations (ii) and (iii) above indicate that the IRA effect is stronger when Home has weaker comparative advantage in the sector. However, as long as $L = L^*$, the IRA effect never dominates the Melitz effect, as each country cannot profitably produce and sell goods in which it does not have sufficiently strong comparative advantage. Without any home market effect, the range of goods produced by Home is limited by its comparative advantage. Therefore, there is still no counter-Melitz effect in any sector.
Next, we allow \( L/ L^* \) to increase above one, in addition to \( a_k \neq 1 \) for some \( k \). Now, because of increasing returns to scale and the home market effect as explained in Krugman (1980), it is possible for the IRA effect to dominate the Melitz effect in Home, as Home is now able to profitably produce goods that it was not able to when \( L = L^* \). In other words, because of its large size, Home is now able to profitably produce goods in which it has comparative disadvantage. In these sectors, \( \frac{d\pi_{ak}}{dB} < 0 \), the IRA effect dominates the Melitz effect, and we have the counter-Melitz effect. However, Foreign, the smaller country, can never have counter-Melitz effect in any sector, because it is not able to profitably produce any good that it was not able to when \( L = L^* \).

As the IRA effects in the comparative advantage sectors are positive, there are gains in real wage in terms of these sectors’ goods upon trade liberalization. Can these gains offset the losses in the comparative disadvantage sectors mentioned above? The answer is, it depends on Home’s relative size. If Home’s relative size is large, and the Foreign-dominated sector is small, then the gains cannot offset the losses. For example, when \( B = 2, L/L^* = 5, \gamma = 2 \) (and therefore \( a_{k_1} = 0.756 \) and \( a_{k_2} = 0.866 \)); and suppose \( a_0 = 0.8 \) (and therefore \( k_1 < 0 \), which means that there does not exist any sector in which Foreign completely dominates). Then, Home will unambiguously lose from trade liberalization, as it loses in the sectors where \( k \in [0, k_2] \), and does not gain or lose in the sectors where \( k \in [k_2, 1] \) and in the homogeneous good sector.

Based on the above analysis, we end this section with the following two testable propositions:

**Proposition 5** Consider the sectors in which both countries produce. For a given country, in the face of trade liberalization, the fraction of exporters increases in the comparative advantage sectors but decreases in the sectors in which the country has the strongest comparative disadvantage, if the country is large compared with the rest of the world.

**Proposition 6** Consider the sectors in which both countries produce. For a given country, in the face of trade liberalization, the fraction of revenue derived from exporting increases in the comparative advantage sectors but decreases in the sectors in which the country has the strongest comparative disadvantage, if the country is large compared with the rest of the world.

### 5.2 Empirical Tests of Propositions 5 and 6

Propositions 5 and 6 predict the existence of a counter-Melitz effect: For a large country, like China, in the sectors where it has the strongest comparative disadvantage but still produces, the fraction of firms that export and the share of revenue derived from exporting will both decrease upon trade liberalization. Can we find any evidence to support the existence of the counter-Melitz effect? This section shows that we indeed find evidence of such an effect.

We test the theory at the 4-digit CIC level, using Chinese industrial firm data from National Bureau of Statistics of China (NBSC), Chinese Customs data from China’s General Administration of Customs
and tariff data from the World Trade Organization (WTO). To get a panel of variables, we need to first tackle the problem caused by a major revision to the Chinese Industry Classification (CIC) in the year 2002. In order to have a consistent definition of sectors, we follow Brandt, Van Biesebroeck & Zhang (2012) by adjusting the 4-digit CIC so as to make the same industry classification code representing the same industry both before and after year 2002.\textsuperscript{15} After adjusting the CIC code for each sector, we aggregate the variables at the 4-digit sector level and obtain a panel of aggregate variables (e.g. mass of firms, mass of firms that export, total revenue, total exporting revenue, etc.) from the years 2001 to 2006, and then calculate the variables we need.\textsuperscript{16}

In order to test the effect of trade liberalization, we also need to establish a proper measure of trade cost $\tau$. As transportation cost is hard to measure and should not vary much in a few years’ time, we take the tariff rate, which decreases a lot after China joined the WTO, as the measure of trade cost. It is also noteworthy that the tariff rates for different sectors are different, which is not consistent with the assumption of our model. Fortunately, it turns out that the results and equations of the model will not be qualitatively affected by the heterogeneity of trade costs across sectors. Therefore, we take into consideration the heterogeneity of tariff rates across sectors in calculating which sectors is predicted to exhibit the counter-Melitz effect according to the theory.

Following Amiti and Konings (2007), Goldberg Khandelwal, Pavcnik and Topalova (2010) and Ge, Lai and Zhu (2011), we construct the industry tariff rate through aggregating tariffs to the 4-digit CIC level. Like the CIC, there was a major recategorization in the international HS 6-digit codes in 2002. Hence we also construct a mapping of the 6-digit HS coding system from the pre-2002 to the post-2002 periods. With these matchings in place, we assign each 8-digit HS product to the 6-digit HS code to which it belongs, and then connect this 6-digits HS code with the standardized 4-digit CIC code, for each year.\textsuperscript{17} Finally, we calculate the industry tariff as the volume-weighted average of all 8-digit HS products which fall into the same 4-digit CIC code for each year. To be precise, the industry tariff is calculated from $\tau_{it} = \left( \sum_{g=1}^{G_i} v_{gt} \tau_{gt} \right) / \left( \sum_{g=1}^{G_i} v_{gt} \right)$, where $\tau_{gt}$ is the 8-digit HS level tariff of an imported good $g$ at year $t$, $v_{gt}$ is the import volume of good $g$ in that year, and $G_i$ is the number of 8-digit HS sectors included in CIC sector $i$. Later, we use these tariff data to test our theory. These industry tariffs are the tariffs faced by the foreign firms entering the Chinese market. For robustness, we use the import volume in 2003 (which is the middle year of our sample), $v_{g2003}$, as the weight for tariff calculation for all years and rerun all the tests. The purpose of doing so is to control for the variance of trade volume within industry across years during trade liberalization period, so that we can get a clearer picture of the pure effect of tariff reduction. In addition, we also allow the elasticity of substitution $\sigma$ and the productivity dispersion level $\gamma$ to be different across sectors. The results listed in Table 1 are qualitatively the same as before. The only change needed in Table 1 is to change $B$ to $B_k = \tau_k^{\gamma_k} \left( \frac{L_k}{F_k} \right)^{\sigma_k^{-1}-1}$. It follows that Propositions 5 and 6 continue to hold. However, different sectors

\textsuperscript{15}The detail of the matching can be found at http://www.econ.kuleuven.be/public/N07057/CHINA/appendix/
\textsuperscript{16}The choice of the years is based on the fact that China acceded to World Trade Organization in December 2001, which further integrate the country with the world. Under the WTO commitments, China cut average tariff from 15.4% in 2001 to 5.31% in 2006.
\textsuperscript{17}The NBSC provide a concordance table between the 6-digit HS code and the 4-digit Chinese industry code. Source of this table can be found at: http://www.5000.gov.cn/release/FrontManage/next_page.aspx?currentPosition=2&cateid=2.
have different threshold for $a_k$: $(a_k)^\gamma < \frac{2B_k}{1 + B_k^2}$, where $B_k = \tau_k^{\gamma_k} \left( \frac{f}{f_x} \right)^{\gamma_k - 1}$. The theory predicts that the counter-Melitz effect will occur in the two-way trade sectors in which the relative productivity $a_k$ is less than $\frac{2B_k}{1 + B_k^2}$. And, the fraction of firms that export in each sector satisfies:

$$\frac{\theta_{xk}}{\theta_{dk}} = \left( \frac{\tau_{dk}}{\tau_{xk}} \right)^\gamma = \frac{1}{B_k} \cdot \frac{B_k (a_k)^{\gamma_k} - 1}{B_k - (a_k)^{\gamma_k}} \cdot \frac{f}{f_x}$$

which is less than $B_k^{1 - 2} f / f_x$ if and only if $(a_k)^\gamma < \frac{2B_k}{1 + B_k^2}$. We calculate the ratio of the fraction of firms that export and $B_k^{1 - 2}$, and rank the twenty-nine 2-digit CIC industries according to this ratio, which we call RATIO. Note that a lower RATIO is assigned a lower ranking. A higher RATIO implies that China has stronger comparative advantage in that sector.\textsuperscript{18} Table 2 shows the ranking of these 2-digit CIC sectors and the corresponding sectoral information that determine the RATIO. We choose the eight sectors with the lowest RATIOs to test for the counter-Melitz effect and the eight sectors with the highest RATIOs to test for the Melitz effect. We run (i) the fraction of exporting firms and (ii) the share of exporting revenue, on the tariff rate for each sector, while controlling for the year and industry (2-digit CIC level) fixed effects and other relevant variables (including employment, capital-labor ratio, average wage in each 4-digit sector). Amongst these variables, employment stands for the size of the sector, and is used to control for the economies of scale; $K/L = \text{capital-labor ratio}$ is used to control for production technique; average wage is used to control for the variable costs and worker skill. The right hand side variables we choose are similar to those of Bernard, Jensen and Schott (2006, Table 4). The results are shown in Tables 3A and 3B. (CA stands for comparative advantage.)

<Table 2 about here>

From Tables 3A and 3B, we see that all the coefficients for the variable “Tariff” are significant in both regressions. Most importantly, the signs are distinctly positive for the eight comparative-disadvantage sectors and distinctly negative for the eight comparative-advantage sectors. In other words, both the fraction of exporting firms and the share of exporting revenue decrease (increase) with trade liberalization for the sectors in which China has comparative disadvantage (comparative advantage). Therefore, we conclude that, consistent with our theory, there exists counter-Melitz effect in the sectors where China has the comparative disadvantage but still produces; while the Melitz effect continues to

\textsuperscript{18} Based on a nested constant-elasticity-substitution utility function, Broda and Weinstein (2006) estimate product-specific elasticities of substitution at the HS 10-digit level. The data is available from Weinstein’s website. We aggregate these numbers up to the HS6 level by taking means, and then use the concordance table from NBSC to calculate the elasticity of substitution of each sector as the average of all the 6-digit HS products which fall into the standardized 2-digit CIC code. For the Pareto location parameter $\gamma_k$, we estimate them using the estimates of $\sigma_k$ and the theoretical prediction that firm sales follow a Pareto distribution with shape parameter $\frac{\gamma_k}{\sigma_k - 1}$ within industry $k$. We follow Eaton, Kortum and Kramarz (2011) in restricting attention to exporters only and back out the shape parameter of the firm sales distribution from a regression of the logarithm of the firm sales rank on the logarithm of firm sales. As Axtell (2001), we find that $\frac{\gamma_k}{\sigma_k - 1}$ is close to 1 for each sector. Hence, we have $B_k \approx \tau_k^{\gamma_k}$ and we use $\tau_k^{\gamma_k}$ to approximate $B_k$, where $\tau_k$ is the average tariff rate in each 2-digit CIC industry and $\gamma_k$ captures the productivity dispersion level in each 2-digit CIC industry.
hold in the sectors in which China has comparative advantage. The coefficients of the other right hand side variables make sense too. For example, higher K/L signifies stronger comparative disadvantage, which lowers exporting ratio and share of export revenue according to our theory. Higher wage signifies higher labor quality, which tends to induce higher propensity to export. Higher employment signifies higher economic scale, which also tends to induce higher propensity to export.

<table>
<thead>
<tr>
<th>Fraction of exporting firms, year 2000-2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regressor</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Tariff</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>log(employment)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>log(K/L)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>log(wage)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Industry fixed effect</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Year fixed effect</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Observations</td>
</tr>
</tbody>
</table>

Note: ***Significant at the 1% level; **Significant at the 5% level; *Significant at the 10% level.

Table 3A

<table>
<thead>
<tr>
<th>Share of exporting revenue in total revenue, year 2000-2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regressor</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Tariff</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>log(employment)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>log(K/L)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>log(wage)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Industry fixed effect</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Year fixed effect</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Observations</td>
</tr>
</tbody>
</table>

Note: ***Significant at the 1% level; **Significant at the 5% level; *Significant at the 10% level.

Table 3B

The choice of the eight sectors with the lowest RATIOs to stand for comparative disadvantage sectors and the eight sectors with the highest RATIOs to stand for comparative advantage sectors may sound a bit arbitrary. Therefore, we check the robustness of the results by varying the set of sectors we choose. We run six regressions for each set of sectors we choose: we run (i) the fraction of exporting firms and (ii)
the share of exporting revenue, on the tariff rate for each set of sectors, while controlling for the year and industry (2-digit CIC level) fixed effects and other relevant variables (including employment, capital-labor ratio, average wage in each 4-digit sector). Each regression is the same as the corresponding one shown in Tables 3A and 3B, though the set of sectors used is different. The result is shown in Figure 5.

![Figure 5.](image)

The horizontal axis in Figure 5 indicates what sectors are included when running the regressions. Note that there is a set of bars on the left and another set of bars on the right. On the left, a number $x$ on the horizontal axis indicates that sectors from the sector with ranking number 1 to the sector with ranking number $x$ are included in the regressions. Therefore, as $x$ increases, the number of comparative disadvantage sectors included in the regressions increase. On the right, a number $x$ indicates that sectors from the sector with ranking number $x$ to the sector with ranking number 29 are included in the regression. Therefore, as $x$ decreases, the number of comparative advantage sectors included in the regressions increase. The vertical axis indicates the number of regressions for which the coefficient for the variable “Tariff” is statistically significant when the corresponding set of sectors indicated on the horizontal axis is included in the regressions. In the figure, the darkest bars represent the number of coefficients with the right sign (positive for the left group of sectors and negative for the right group of sectors) and significant at 1% level. The second darkest bars represent the number of coefficients with the right sign and significant at 5% level (but not significant at 1% level). The lightest bars represent the number of coefficients with the right sign, but not significant at 5% level.\(^{19}\) From the figure, it is

\(^{19}\)For example, a number 9 on the horizontal axis indicates that sectors with rankings 1 through 9 are included in the regressions. Corresponding to that, all six regressions similar to the ones shown in Table 3A and 3B yield positive and
clear that the counter-Melitz effect becomes significant when we include sufficiently large number of sectors with the smallest RATIOs (the figure shows that 4 is a sufficiently large number of sectors), and the effect remains significant till we include the 9 sectors with the smallest RATIOs. Likewise, the Melitz effect becomes significant when we include sufficiently large number of sectors with the largest RATIOs (the figure shows that 4 is also a sufficiently large number of sectors), and the effect remains significant till we include the 11 sectors with the highest RATIOs. The coefficients for the sectors at both ends of the ranking are mostly not very significant, probably due to the limited sample size (too few observations). The coefficients are mostly not very significant when the sectors in the middle of the ranking are included. This is consistent with our theory, as they are sectors at the margin, and neither the Melitz effect nor the counter-Melitz effect dominate. Thus, the total effect is ambiguous.

Table 4 shows the results of the OLS regressions when the data of all sectors are pooled together.

<table>
<thead>
<tr>
<th>Year 2000-2006</th>
<th>Fraction of exporting firms</th>
<th>Share of exporting revenue in total revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regressor</td>
<td>All sectors</td>
<td>All sectors</td>
</tr>
<tr>
<td>Tariff</td>
<td>-0.010</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>log(employment)</td>
<td>0.016***</td>
<td>0.013***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>log(K/L)</td>
<td>-0.112***</td>
<td>-0.142***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>log(wage)</td>
<td>0.124***</td>
<td>0.107***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Industry fixed effect</td>
<td>NA</td>
<td>Yes</td>
</tr>
<tr>
<td>Year fixed effect</td>
<td>NA</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>2430</td>
<td>2430</td>
</tr>
</tbody>
</table>

Note: ***Significant at the 1% level; **Significant at the 5% level; *Significant at the 10% level.

Table 4

In Table 4, most of the coefficients are not statistically significant and even the signs of the coefficients are ambiguous. This is consistent with our theory, as some sectors exhibit counter-Melitz effect while others exhibit Melitz effect, and therefore the total effect maybe ambiguous and statistically insignificant. This result contrast with that obtained by Bernard, Jensen & Schott (2006). They use plant level data of the U.S. and run a similar regression of the probability of exporting on change in trade cost. They get negative sign (Melitz effect) at 10% level of significance for that regression. The main reason for the difference might be that, unlike China, the U.S. does not have many sectors in which it has strong comparative disadvantage but in which it still produces. As a result, fewer plants in the U.S. would show counter-Melitz effect. Therefore, the overall outcome is dominated by the Melitz effect. Moreover, significant coefficients (at 1% level) for variable “Tariff”. A number 21 on the horizontal axis indicates that sectors with rankings 21 through 29 are included in the regressions. Corresponding to that, five of the regressions similar to the ones shown in Table 3A and 3B yield negative and significant coefficients (at 1% level) for “Tariff”, while one regression yields negative and significant coefficient at 5% level for “Tariff”.

Some 2-digit industries contain fewer than ten 4-digit sectors. Thus the degree of freedom of the regression may be limited if we choose too few 2-digit industries for testing our propositions.
the coefficient is also not very significant statistically as the Melitz effect is countered by the IRA effect in some sectors.

**Robustness of the Result**

For robustness check, we construct the tariff for each industry using the import volume in 2003 (which is the middle year of our sample), $v_{2003}$, as the weight for tariff calculation for all years and report the regression results in Figure 6. As in Figure 5, Figure 6 shows that the counter-Melitz effect becomes significant when we include sufficiently large number of sectors with the smallest RATIOs, and the effect remains significant until we include the 7 sectors with the smallest RATIOs. Likewise, the Melitz effect becomes significant when we include sufficiently large number of sectors with the largest RATIOs, and the effect remains significant until we include the 16 sectors with the highest RATIOs. Thus, these results are also consistent with the predictions of our theory. Furthermore, we report the results of the OLS regressions when the data of all sectors are pooled together in Table 5. Like in Table 4, the coefficients in Table 5 are not statistically significant either, as the counter-Melitz effect and the Melitz effect cancel each other when all sectors are pooled together.

![Figure 6. Robustness of Empirical Results](image-url)
6 Can firms that sell domestically be more productive than firms that don’t?

So far, we simplified our analysis by assuming that $\frac{f}{f_x} > \max\{\frac{f}{L}, \frac{f}{L^*}\}$ so as to exclude the possibility that some firms only export but do not serve the domestic market. In fact, our model can accommodate this possibility, which is consistent with the recent finding based on Chinese data by Lu (2010), that in some industries, the exporters in China have a relative lower average productivity compared with the firms that serve the domestic market.

First of all, we adopt the assumption that a firm needs to incur a market entry cost if it enters a market, be it domestic or foreign. In this case, $f$ and $f_x$ stand for the amortized fixed market-entry cost per period plus the overhead cost per period for serving the domestic market and foreign market respectively. In this case, we can relax the assumption $\frac{f}{f_x} > \max\{\frac{f}{L}, \frac{f}{L^*}\}$ (only $\tau^{-1} f_x > f$ is needed), and all of our propositions still hold. The main point is that it is possible that for some sector $k$, we have $\overline{\tau}_{dk} > \overline{\tau}_{xk}$. Once we have this outcome, we can have the situation where some firms in the sectors where a country has the strongest comparative advantage may only export; thus in these sectors the firms that only export are less productive than those that also serve the domestic market. This is also consistent with Dan Lu’s finding that in labor-intensive sectors, firms that serve domestic market have higher productivity than exporters, as China has comparative advantage in the labor-intensive industries.

Detailed calculation is given in the appendix. We state our result in

**Proposition 7** If the fixed entry cost for exporting is not too high, then in the sectors where a country

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21 This is different from the original assumption in Melitz (2003) as well in this paper so far.
has the strongest comparative advantage, the firms that do not serve the domestic market can have a lower average productivity than those that do.

Note that the assumption that the fixed cost of entry for exporting is relatively low compared with the entry cost in the domestic market fits the observation that the cost of entry into processing trade in China is relatively low because of government policy. Firms engaging in processing trade enjoy tariff concessions whereas firms that sell domestically do not. In fact, Dai, Maitra and Yu (2011) find that Lu’s result is primarily driven by the existence of processing trade in the labor-intensive sectors of China. They find that once processing trade is excluded from the analysis, exporting firms are more productive than non-exporting firms in all sectors, even in China.

7 Conclusion

In this paper, we merge the heterogenous firm model of Melitz (2003) with the Ricardian model of Dornbusch et al (1977) to form a hybrid model to explain how the pattern of international specialization and trade is determined by the interaction of comparative advantage, economies of scale, country sizes and trade barriers. The model is able to capture the existence of inter-industry trade and intra-industry trade in a single unified framework. It explains how trade openness and trade liberalization affects the pattern of international specialization and trade. The paper examines the generality of Melitz’s firm selection effect in response to trade liberalization in a multi-sectoral setting.

Although opening to trade is welfare-improving in both countries, trade liberalization can lead to a counter-Melitz effect in the larger country in the sectors in which it has the strongest comparative disadvantage but in which it still produces. In this case, the productivity cutoff for survival is lowered while the exporting cutoff increases in the face of trade liberalization, leading to reductions in real wage in terms of these goods. This is because the inter-sectoral resource allocation (IRA) effect dominates the Melitz effect in these sectors. The larger country can profitably produce in the comparative disadvantage sectors because of increasing returns to scale and home market effect. For this reason the counter-Melitz effect does not exist in the smaller country. Empirical evidence in the years 2000-2006 confirms that the fraction of exporting firms as well as the share of export revenue in total revenue both decreased in the comparative-disadvantage sectors of China in the face of trade liberalization, consistent with our hypothesis.

Our basic model can be easily modified to explain both the existence of the counter-Melitz effect and the peculiar features of some Chinese labor-intensive sectors as found by Lu (2010) and others.
Appendixes

A Solving for the System

In this appendix, we will show how to solve the model for the sectors where both countries produce. In other words, we solve for \((\varphi_{dk}, \varphi_{dk}^*, \varphi_{xk}, \varphi_{xk}^*, \theta_{dk}, \theta_{dk}^*)\) from the system constituted of the four zero cutoff profit conditions and two free entry conditions. Combining the two zero cutoff conditions for firms serving the Home market, (11) and (14), we have

\[
\frac{\varphi_{xk}^*}{\varphi_{dk}} = a_k \left( \frac{Bf_x}{f} \right)^{1/n} \tag{31}
\]

Similarly, combining those for firms serving Foreign’s market, (12) and (13), we can get

\[
\frac{\varphi_{xk}}{\varphi_{dk}^*} = \frac{1}{a_k} \left( \frac{Bf_x}{f} \right)^{1/n} \tag{32}
\]

Equations (31), (32), and the FE conditions (15), and (16) now form a system of four equations and four unknowns, \(\varphi_{dk}, \varphi_{xk}, \varphi_{dk}^*, \varphi_{xk}^*\). Solving, we obtain (17), (18), (19) and (20).

Then recall that the aggregate price indexes are given by

\[
P_k = \frac{1}{1 - \sigma} p_{dk}(\varphi_k)
\]

and

\[
P_k^* = (\theta_k^*)^{-1} \frac{1}{1 - \sigma} p_{dk}^*(\varphi_k^*)
\]

Substituting these price indexes into Zero Cutoff Conditions (11) and (12), and with the help of equation (9) and (10), we have

\[
\sigma f = \frac{b_k L}{\theta_k} \left( \frac{\varphi_{dk}}{\varphi_k} \right)^{\sigma - 1} - \frac{1}{\gamma} \frac{b_k L}{\theta_{dk} + \theta_{xk} \frac{f}{f_x}} \tag{33}
\]

\[
\sigma f = \frac{b_k L^*}{\theta_k^*} \left( \frac{\varphi_{dk}^*}{\varphi_k^*} \right)^{\sigma - 1} - \frac{1}{\gamma} \frac{b_k L^*}{\theta_{dk}^* + \theta_{xk}^* \frac{f}{f_x}} \tag{34}
\]

From the equilibrium productivity cutoffs (17) and (18) in both countries, we get

\[
\left( \frac{\varphi_{dk}}{\varphi_{dk}^*} \right)^{\gamma} = \frac{B - (a_k)^{-\gamma}}{B - (a_k)^{\gamma}} \tag{35}
\]

Therefore, the number of exporting firms in Home and Foreign are respectively:

\[
\theta_{xk} = \left( \frac{\varphi_{dk}}{\varphi_{xk}} \right)^{\gamma} \theta_{dk} = \left( \frac{a_k \varphi_{dk}}{\varphi_{xk}} \right)^{\gamma} \left( \frac{f}{Bf_x} \right) \theta_{dk} \tag{36}
\]

\[
\theta_{xk}^* = \left( \frac{\varphi_{dk}^*}{\varphi_{xk}^*} \right)^{\gamma} \theta_{dk}^* = \left( \frac{\varphi_{dk}^*}{a_k \cdot \varphi_{dk}} \right)^{\gamma} \left( \frac{f}{Bf_x} \right) \theta_{dk}^* \tag{37}
\]

Equations (33), (34), (35), (36), (37) then imply (21) and (22).

\(\theta_{xk}\) and \(\theta_{xk}^*\) can be obtained by substituting (35), (21), (22) into (36) and (37) respectively.
B Welfare Impact of Trade Liberalization

In this appendix, we will prove how the real wage in terms of the aggregate good of sector k (thereafter called real wage in terms of good k) changes after trade liberalization in three cases. Without loss of generality, we assume that $L > L^*.$

1. Foreign-dominated sectors: $k \in (0, k_1)$. The real wage in terms of good k in this zone in Home and Foreign are, respectively:

$$\frac{1}{P_k} = \left(\frac{\theta_{xk}}{\sigma - 1}\right)^{\frac{1}{\sigma - 1}} \rho A_k \varphi_{xk} \frac{1}{\tau} = \rho A_k^* B^{-\frac{1}{\gamma}} \left(\frac{L + L^*}{L} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma - \sigma + 1} D_2 (k) L\right)^{\frac{1}{\sigma - 1}}$$

$$\frac{1}{P_k^*} = \left(\frac{\theta_{dk}}{\sigma - 1}\right)^{\frac{1}{\sigma - 1}} \rho A_k^* \varphi_{dk} = \rho A_k^* \left(\frac{L + L^*}{L^*} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma - \sigma + 1} D_2 (k) L^*\right)^{\frac{1}{\sigma - 1}}$$

Since trade liberalization will increase $\frac{1}{P_k}$ as $B$ falls, the real wage in terms of good k in Home will be improved. However, the real wage in Foreign, $\frac{1}{P_k^*}$, is not related to the trade barriers. That’s, trade liberalization does not affect the real wage in Foreign.

2. Both countries produce: $k \in (k_1, k_2)$. The real wage in Home and Foreign in terms of good k are equal to:

$$\frac{1}{P_k} = \left(\frac{\theta_{ck}}{\sigma - 1}\right)^{\frac{1}{\sigma - 1}} \rho A_k \varphi_{dk} = \rho A_k \left(D_1 \frac{B - B^{-1}}{B - (a_k)^{\gamma}} \right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma - \sigma + 1} D_2 (k) L\right)^{\frac{1}{\sigma - 1}}$$

$$\frac{1}{P_k^*} = \left(\frac{\theta_{ck}}{\sigma - 1}\right)^{\frac{1}{\sigma - 1}} \rho A_k^* \varphi_{dk} = \rho A_k^* \left(D_1 \frac{B - B^{-1}}{B - (a_k)^{\gamma}} \right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma - \sigma + 1} D_2 (k) L^*\right)^{\frac{1}{\sigma - 1}}$$

This zone is divided into two cases:

(a) Scenario A: $(a_k)^\gamma < \frac{2B}{1+B^\gamma}.$

Note that $\frac{B - B^{-1}}{B - (a_k)^{\gamma}}$ decreases but $\frac{B - B^{-1}}{B - (a_k)^{\gamma}}$ increases as trade barrier $B$ falls, as $(a_k)^\gamma < \frac{2B}{1+B^\gamma}.$ Therefore, the real wage in terms of good k in Home will decline, but the real wage in Foreign rises. This is the case with counter-Melitz effect in Home.

(b) Scenario B: $(a_k)^\gamma \in \left(\frac{2B}{1+B^\gamma}, \frac{1+B^2}{2B}\right).$

Since both $\frac{B - B^{-1}}{B - (a_k)^{\gamma}}$ and $\frac{B - B^{-1}}{B - (a_k)^{\gamma}}$ increase as trade barrier $B$ falls when $(a_k)^\gamma \in \left(\frac{2B}{1+B^\gamma}, \frac{1+B^2}{2B}\right),$ the real wages in terms of good k in both countries increase in this zone.

3. Home-dominated sectors: $k \in (k_2, 1)$. Real wages in terms of good k are given by

$$\frac{1}{P_k} = \left(\frac{\theta_{xk}}{\sigma - 1}\right)^{\frac{1}{\sigma - 1}} \rho A_k \varphi_{xk} = \rho A_k B^{-\frac{1}{\gamma}} \left(\frac{L + L^*}{L} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma - \sigma + 1} D_2 (k) L\right)^{\frac{1}{\sigma - 1}}$$

$$\frac{1}{P_k^*} = \left(\frac{\theta_{xk}}{\sigma - 1}\right)^{\frac{1}{\sigma - 1}} \rho A_k \varphi_{xk} = \rho A_k B^{-\frac{1}{\gamma}} \left(\frac{L + L^*}{L^*} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma - \sigma + 1} D_2 (k) L^*\right)^{\frac{1}{\sigma - 1}}$$

It is clear that real wage in terms of good k in Home is unchanged but that in Foreign increases as $B$ falls.
C When the assumption $\frac{f_x}{f} > \max\{\frac{L}{L^*}, \frac{L^*}{L}\}$ is relaxed

Table 1 shows that that $\varphi_{dk} > \varphi_{xk}$ is equivalent to $\frac{f}{f_x} < \frac{L}{L^*}$ in the Home-dominated sectors $k \in (k_2, 1]$. In the Foreign-dominated sectors $k \in [0, k_1)$, $\varphi_{dk}^* > \varphi_{xk}^*$ is equivalent to $\frac{f}{f_x} < \frac{L}{L^*}$. In the two-way trade sectors $k \in (k_1, k_2)$, $\varphi_{dk} > \varphi_{xk}$ is equivalent to $\frac{B^2 f_x + 1}{B \left(\frac{L}{f_x} + 1\right)}$ and $\varphi_{dk}^* > \varphi_{xk}^*$ is equivalent to $(a_k)^\gamma < \frac{B \left(\frac{L}{f_x} + 1\right)}{B^2 \left(\frac{L}{f_x} + 1\right)}$. Here, we can introduce two new thresholds $k_3$ and $k_4$, such that, $(a_{k_3})^{\gamma} = \frac{B^2 f_x + 1}{B \left(\frac{L}{f_x} + 1\right)}$ and $(a_{k_4})^{\gamma} = \frac{B \left(\frac{L}{f_x} + 1\right)}{B^2 \left(\frac{L}{f_x} + 1\right)}$.

It is clear that the assumption $\frac{f}{f_x} > \max\{\frac{L}{L^*}, \frac{L^*}{L}\}$ ensures that $k_3 < k_1 < k_2 < k_4$. Thus $\varphi_{dk} < \varphi_{xk}$ and $\varphi_{dk}^* < \varphi_{xk}^*$ in all sectors $k \in [0, 1]$, and all firms either only serve the domestic market, or export and sell to domestic market at the same time. If the assumption $\frac{f}{f_x} > \max\{\frac{L}{L^*}, \frac{L^*}{L}\}$ is relaxed, some Home producers can only sell to the Foreign market if $\frac{f}{f_x} < \frac{L}{L^*}$ (which is equivalent to $k_4 < k_2$). Furthermore, some Foreign producers can only sell to Home’s market if $\frac{f}{f_x} < \frac{L}{L^*}$ (which is equivalent to $k_1 < k_3$).

Now relax the assumption $\frac{f}{f_x} > \max\{\frac{L}{L^*}, \frac{L^*}{L}\}$ while maintaining the assumption $\tau^{\sigma-1} f_x > f$ (which ensures that some firms produce exclusively for their domestic market in both countries in some sectors, i.e., $k_3 < k_4$). Under the condition that $L^* < L$, we have the following three cases:

(a): If $\frac{L^*}{L} < \frac{f}{f_x} < \frac{L}{L^*}$ and $B^2 \frac{L}{f_x} \frac{L^*}{L} > 1$, then we have $k_1 < k_3 < k_2 < k_4$.

(b): If $\frac{L^*}{L} < \frac{f}{f_x} < \frac{L}{L^*}$ and $B^2 \frac{L}{f_x} \frac{L^*}{L} < 1$, then we have $k_1 < k_2 < k_3 < k_4$.

(c): If $\frac{f}{f_x} < \min\{\frac{L}{L^*}, \frac{L^*}{L}\}$, then we have $k_1 < k_3 < k_4 < k_2$.

In all those three cases, there are some sectors in which the firms in some country that do not serve the domestic market can have a lower average productivity than those that do. In the following discussion, we focus our attention on case (c), i.e., $k_1 < k_3 < k_4 < k_2$. In zone $[0, k_1)$, only Foreign’s firms produce; some of them only export, while others serve both markets. In zone $(k_1, k_3)$, firms in both countries produce; Home’s firms either serve both markets or Home’s market only, while Foreign’s firms either only export or sell to both markets. In zone $(k_3, k_4)$, firms in both countries produce; they either serve both markets or their own domestic market only. In zone $(k_4, k_2)$, firms in both countries produce; Home’s firms either only export or sell to both markets, while Foreign’s firms either serve both markets or their domestic market only. In zone $(k_2, 1)$, only Home’s firms produce; some of them only export, and some serve both markets. Consequently, in zone $(k_4, 1)$, where Home has comparative advantage, we have $\varphi_{dk} > \varphi_{xk}$, provided that $\frac{1}{\tau^{\sigma-1}} < \frac{f}{f_x} < \min\{\frac{L}{L^*}, \frac{L^*}{L}\}$. 

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References


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Note: Columns 6-11 represent the mean of the tariff, exporting ratio, total revenue, total exporting revenue, firm number and total employment within each industry across years.